

Problem 10.1 In Active Example 10.1, suppose that the distance from point A to point C is increased from $\frac{1}{4}L$ to $\frac{1}{2}L$. Draw a sketch of the beam with C in its new position. Determine the internal forces and moment at C.

Solution: The reactions at A and B are repeated from Active Example 10.1.

Passing a plane through the beam at point C and writing the equilibrium equations for the part of the beam to the left of C, we obtain

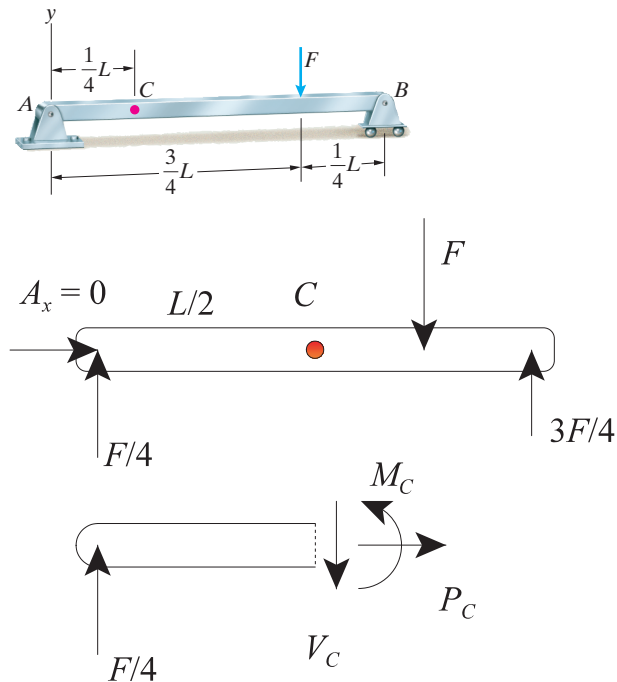
$$\Sigma F_x : P_C = 0,$$

$$\Sigma F_y : \frac{1}{4}F - V_C = 0,$$

$$\Sigma M_C : M_C - (\frac{1}{2}L)(\frac{1}{4}F) = 0.$$

Solving yields

$$P_C = 0, V_C = \frac{1}{4}F, M_C = \frac{1}{8}LF.$$



Problem 10.2 The magnitude of the triangular distributed load is $w_0 = 2 \text{ kN/m}$. Determine the internal forces and moment at A.

Solution: The free-body diagram of the beam is shown in Fig. a. From the equilibrium equations

$$\Sigma F_x : B_x = 0,$$

$$\Sigma F_y : B_y + C - 600 \text{ N} = 0,$$

$$\Sigma M_B : C(1.2 \text{ m}) - (600 \text{ N})(1 \text{ m}) = 0,$$

We obtain

$$B_x = 0, B_y = 100 \text{ N}, C = 500 \text{ N}.$$

Passing a plane through the beam at A and isolating the part of the beam to the left of A (Fig. b), we obtain

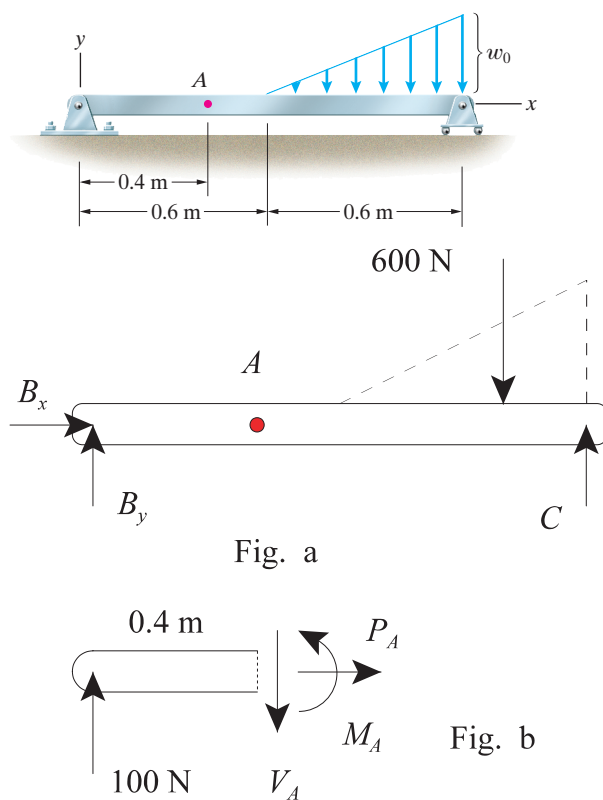
$$\Sigma F_x : P_A = 0,$$

$$\Sigma F_y : 100 \text{ N} - V_A = 0,$$

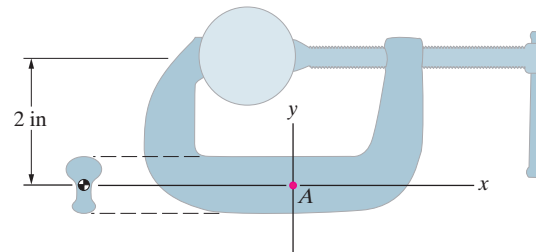
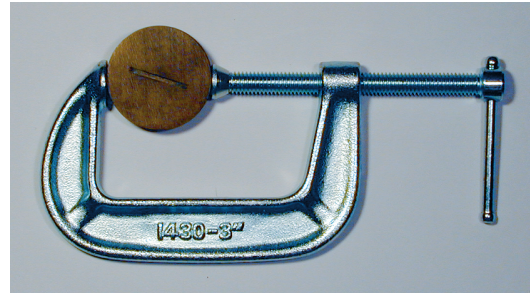
$$\Sigma M_A : M_A - (100 \text{ N})(0.4 \text{ m}) = 0.$$

Solving yields

$$P_A = 0, V_A = 100 \text{ N}, M_A = 40 \text{ N}\cdot\text{m}.$$



Problem 10.3 The C clamp exerts 30-lb forces on the clamped object. Determine the internal forces and moment in the clamp at A.



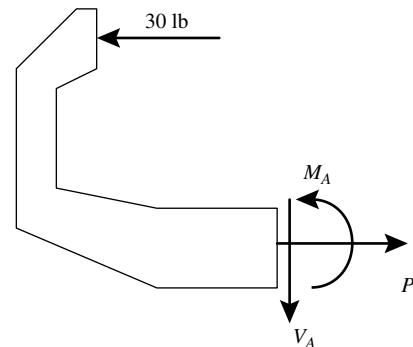
Solution:

$$\sum F_x : -30 \text{ lb} + P_A = 0$$

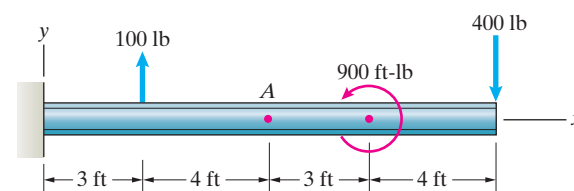
$$\sum F_y : -V_A = 0$$

$$\sum M_A : (30 \text{ lb})(2 \text{ in}) + M_A = 0$$

Solving: $P_A = 30 \text{ lb}, V_A = 0, M_A = -60 \text{ in-lb}$



Problem 10.4 Determine the internal forces and moment at A.



Solution: Passing a plane through the beam at A and writing the equilibrium equations for the part of the beam to the right of A, we obtain

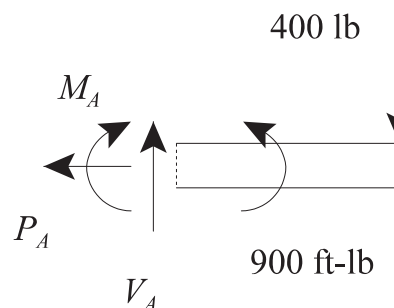
$$\sum F_x : -P_A = 0,$$

$$\sum F_y : V_A - 400 \text{ lb} = 0,$$

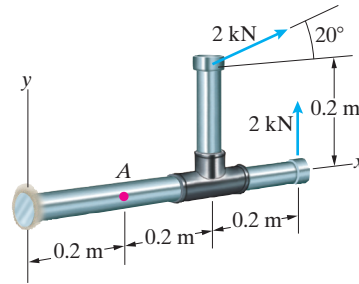
$$\sum M_A : -M_A + 900 \text{ ft-lb} - (400 \text{ lb})(7 \text{ ft}) = 0.$$

Solving yields

$P_A = 0, V_A = 400 \text{ lb}, M_A = -1900 \text{ ft-lb}.$



Problem 10.5 The pipe has a fixed support at the left end. Determine the internal forces and moment at A.



Solution: Use the right section

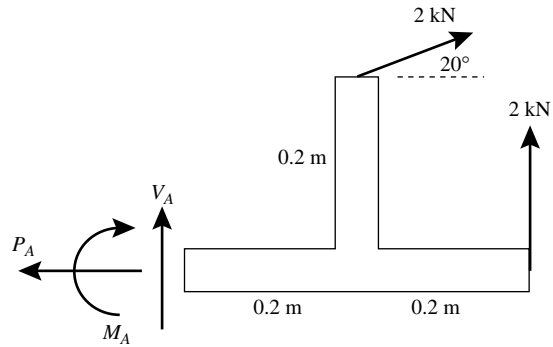
$$\sum F_x : -P_A + 2 \text{ kN} \cos 20^\circ = 0$$

$$\sum F_y : V_A + 2 \text{ kN} \sin 20^\circ + 2 \text{ kN} = 0$$

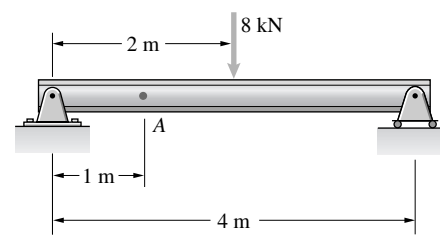
$$\begin{aligned} \sum M_A : -M_A - (2 \text{ kN} \cos 20^\circ)(0.2 \text{ m}) \\ + (2 \text{ kN} \sin 20^\circ)(0.2 \text{ m}) + (2 \text{ kN})(0.4 \text{ m}) = 0 \end{aligned}$$

Solving:

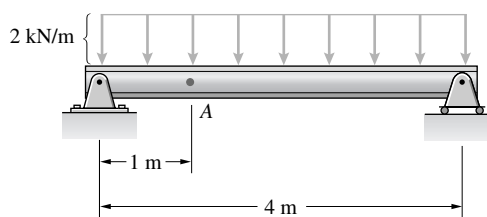
$$P_A = 1.88 \text{ kN}, V_A = -2.68 \text{ kN}, M_A = 0.561 \text{ kN-m}$$



Problem 10.6 Determine the internal forces and moment at A for each loading.



(a)



(b)

Solution: (a) Denote the reaction at the pinned left end by R , and the reaction at the roller support by B . The reaction at B :

$$\sum M = -2(8) + B(4) = 0,$$

from which $B = 4$ kN. The reaction at R :

$$\sum F_y = R_y - 8 + B = 0,$$

from which $R_y = 4$ kN.

$$\sum F_x = R_x = 0.$$

Make a cut at A: Isolate the left hand part. The sum of moments:

$$\sum M = M_A - 4(1) = 0,$$

from which $M_A = 4$ kN-m $V_A = 4$ kN $P_A = 0$

(b) Determine the reaction at B : The sum of the moments about R :

$$\sum M_R = - \int_0^4 2x \, dx + 4B = 0,$$

from which

$$B = \left(\frac{1}{4} \right) \left[2 \frac{x^2}{2} \right]_0^4 = \frac{16}{4} = 4 \text{ kN}.$$

The reaction at R :

$$\sum F_y = R_y - \int_0^4 2 \, dx + B = 0,$$

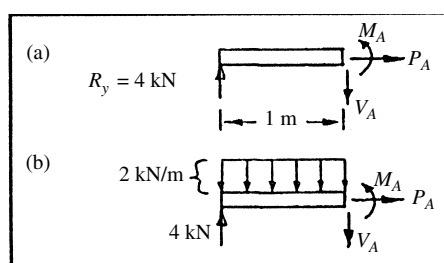
from which

$$R_y = 8 - 4 = 4 \text{ kN},$$

$$\sum F_x = R_x = 0.$$

Make a cut at A: Isolate the left hand part. The sum of moments:

$$\sum M = M_A - (1)R_y + \int_0^1 2x \, dx = 0,$$

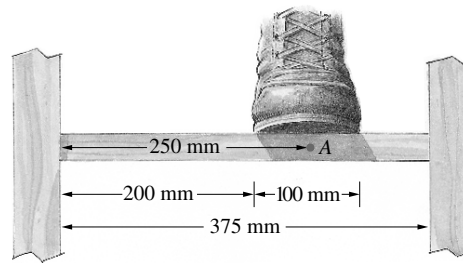


from which $M_A = R_y - 1 = 3$ kN-m.

$$V_A = R_y - \int_0^1 2 \, dx = 4 - 2 = 2 \text{ kN}$$

$P_A = 0$.

Problem 10.7 Model the ladder rung as a simply supported (pin supported) beam and assume that the 750-N load exerted by the person's shoe is uniformly distributed. Determine the internal forces and moment at A.

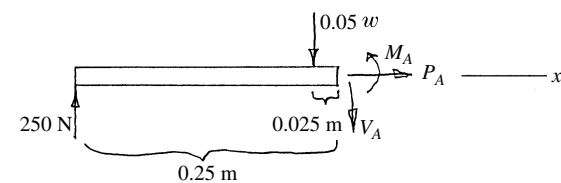
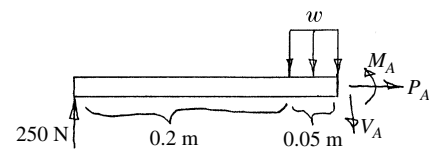
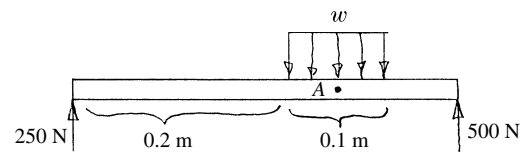
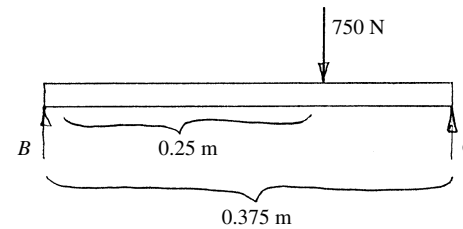


Solution:

$$\sum F_y = B + C - 750 = 0,$$

$$\sum M_{(\text{pt. } B)} = 0.375C - (0.25)(750) = 0.$$

Solving, $B = 250 \text{ N}$, $C = 500 \text{ N}$.



The distributed load is

$$w = (750 \text{ N}) / (0.1 \text{ m}) = 7500 \text{ N/m}.$$

From the equilibrium equations

$$\sum F_x = P_A = 0,$$

$$\sum F_y = 250 - V_A - 0.05(7500) = 0,$$

$$\sum M_{(\text{rightend})} = M_A - (250)(0.25)$$

$$+ (0.05)(7500)(0.025) = 0,$$

we obtain $P_A = 0$, $V_A = -125 \text{ N}$, $M_A = 53.1 \text{ N-m}$.

Problem 10.8 In Example 10.2, suppose that the distance from point A to point B is increased from 3 m to 4 m. Draw a sketch of the beam with B in its new position. Determine the internal forces and moment at B .

Solution: Let us pass a plane through the beam at B . Using similar triangles, the magnitude of the distributed load at B is

$$\frac{4}{6}(60 \text{ N/m}) = 40 \text{ N/m}.$$

If we represent the distributed load to the left of B by an equivalent force, its magnitude is

$$\frac{1}{2}(40 \text{ N/m})(4 \text{ m}) = 80 \text{ N},$$

and it acts at the centroid of the distributed load to the left of B . The distance from A to the centroid is

$$\frac{2}{3}(4 \text{ m}) = 2.67 \text{ m}.$$

The equilibrium equations for the part of the beam to the left of B are

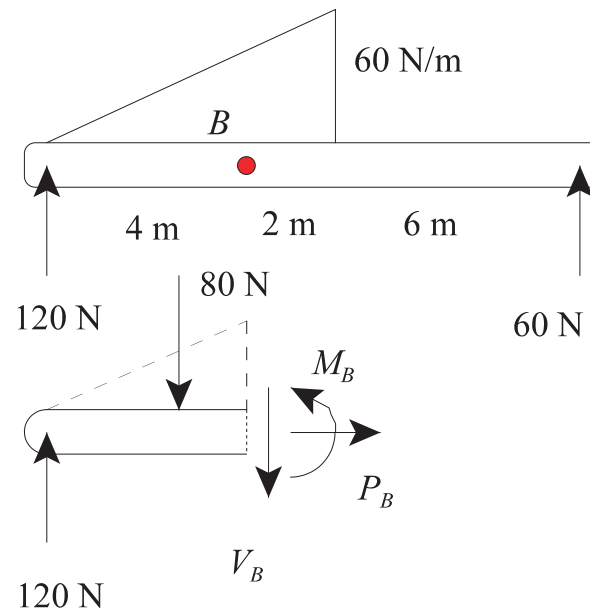
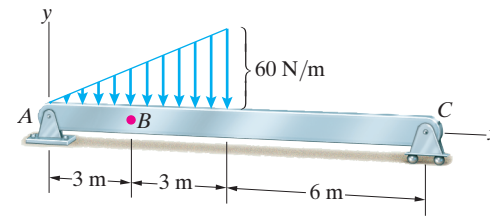
$$\Sigma F_x : P_B = 0,$$

$$\Sigma F_y : 120 \text{ N} - 80 \text{ N} - V_B = 0,$$

$$\Sigma M_B : M_B - (120 \text{ N})(4 \text{ m}) + (80 \text{ N})(1.33 \text{ m}) = 0.$$

Solving yields

$$P_B = 0, V_B = 40 \text{ N}, M_B = 373 \text{ N}\cdot\text{m}.$$



Problem 10.9 If $x = 3$ ft, what are the internal forces and moment at A ?

Solution: Isolating the part of the beam to the right of A , we represent the distributed load by an equivalent force. From this free-body diagram, we write the equilibrium equations:

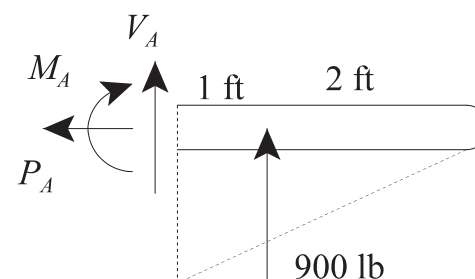
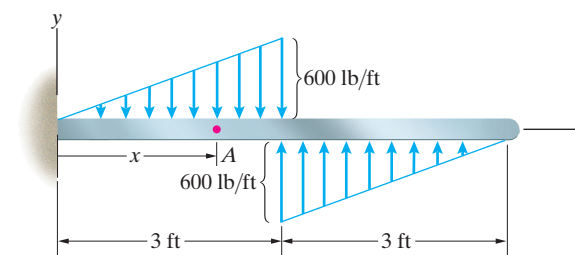
$$\Sigma F_x : -P_A = 0,$$

$$\Sigma F_y : V_A + 900 \text{ lb} = 0,$$

$$\Sigma M_A : -M_A + (900 \text{ lb})(1 \text{ ft}) = 0.$$

Solving yields

$$P_A = 0, V_A = -900 \text{ lb}, M_A = 900 \text{ ft}\cdot\text{lb}.$$



Problem 10.10 If $x = 4$ ft, what are the internal forces and moment at A?

Solution: Isolating the part of the beam to the right of A, we represent the distributed load by an equivalent force.

We can obtain the magnitude of the distributed load by similar triangles:

$$\frac{2}{3}(600 \text{ lb/ft}) = 400 \text{ lb/ft.}$$

If we represent the distributed load to the right of point A by a single equivalent force, its magnitude is

$$\frac{1}{2}(400 \text{ lb/ft})(2 \text{ ft}) = 400 \text{ lb,}$$

and it acts at the centroid of the distributed load to the right of point A. The distance from A to the centroid is

$$\frac{1}{3}(2 \text{ ft}) = 0.667 \text{ ft.}$$

From this free-body diagram, we write the equilibrium equations:

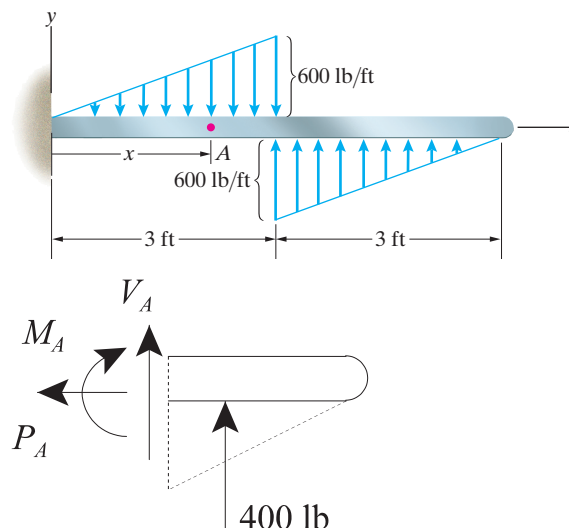
$$\Sigma F_x : -P_A = 0,$$

$$\Sigma F_y : V_A + 400 \text{ lb} = 0,$$

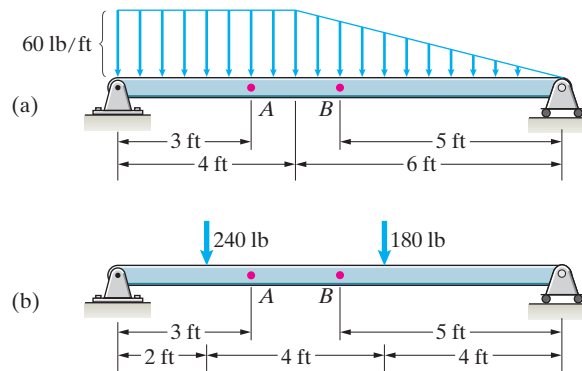
$$\Sigma M_A : -M_A + (400 \text{ lb})(0.667 \text{ ft}) = 0.$$

Solving yields

$$P_A = 0, V_A = -400 \text{ lb}, M_A = 267 \text{ ft-lb.}$$



Problem 10.11 Determine the internal forces and moment at A for the loadings (a) and (b).

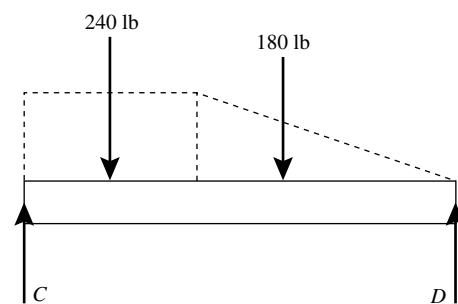


Solution: The external reactions are the same for either loading condition

$$\sum M_A : -(240 \text{ lb})(2 \text{ ft}) - (180 \text{ lb})(6 \text{ ft}) + D(10 \text{ ft}) = 0$$

$$\Rightarrow D = 156 \text{ lb}$$

$$\sum F_y : C + D - 240 \text{ lb} - 180 \text{ lb} = 0 \Rightarrow C = 264 \text{ lb}$$



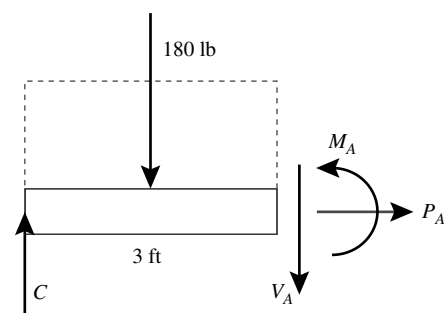
(a) Use the left section with the distributed loading

$$\sum F_x : P_A = 0$$

$$\sum F_y : C - 180 \text{ lb} - V_A = 0$$

$$\sum M_A : -C(3 \text{ ft}) + (180 \text{ lb})(1.5 \text{ ft}) + M_A = 0$$

$$\text{Solving } P_A = 0, V_A = 84 \text{ lb}, M_A = 522 \text{ ft-lb}$$



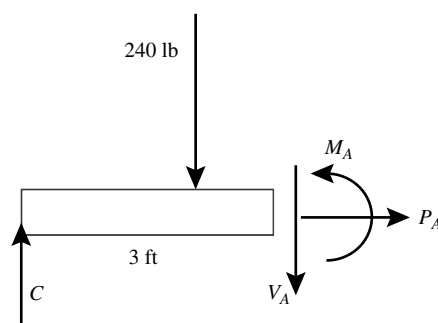
(b) Use the left section with the discrete load

$$\sum F_x : P_A = 0$$

$$\sum F_y : C - 240 \text{ lb} - V_A = 0$$

$$\sum M_A : -C(3 \text{ ft}) + (240 \text{ lb})(1.0 \text{ ft}) + M_A = 0$$

$$\text{Solving } P_A = 0, V_A = 24 \text{ lb}, M_A = 552 \text{ ft-lb}$$



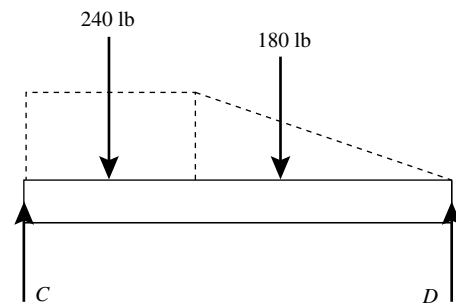
Problem 10.12 For the loadings (a) and (b) shown in Problem 10.11, determine the internal forces and moment at B .

Solution: The external reactions are the same for either loading condition

$$\sum M_A : -(240 \text{ lb})(2 \text{ ft}) - (180 \text{ lb})(6 \text{ ft}) + D(10 \text{ ft}) = 0$$

$$\Rightarrow D = 156 \text{ lb}$$

$$\sum F_y : C + D - 240 \text{ lb} - 180 \text{ lb} = 0 \Rightarrow C = 264 \text{ lb}$$



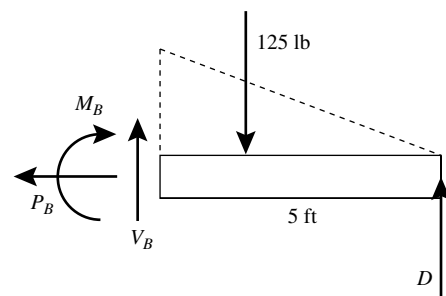
(a) Use the right section with the distributed loading

$$\sum F_x : -P_B = 0$$

$$\sum F_y : V_B - 125 \text{ lb} + D = 0$$

$$\sum M_B : -M_B - 125 \text{ lb} \left(\frac{1}{3} 5 \text{ ft} \right) + D(5 \text{ ft}) = 0$$

Solving $P_B = 0, V_B = -31 \text{ lb}, M_B = 572 \text{ ft-lb}$



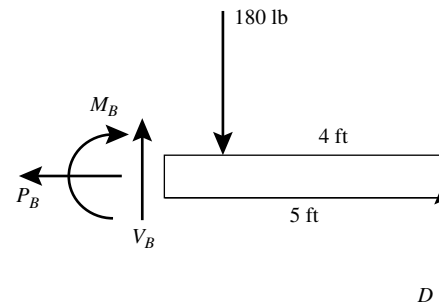
(b) Use the right section with the discrete loads

$$\sum F_x : -P_B = 0$$

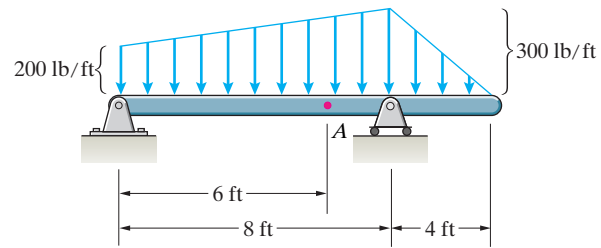
$$\sum F_y : V_B - 180 \text{ lb} + D = 0$$

$$\sum M_B : -M_B - (180 \text{ lb})(1 \text{ ft}) + D(5 \text{ ft}) = 0$$

Solving $P_B = 0, V_B = 24 \text{ lb}, M_B = 600 \text{ ft-lb}$

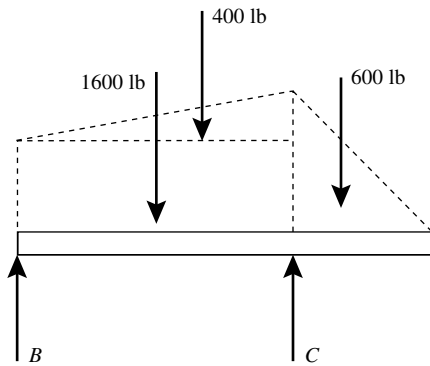


Problem 10.13 Determine the internal forces and moment at A.



Solution: Use the whole body to find the reactions

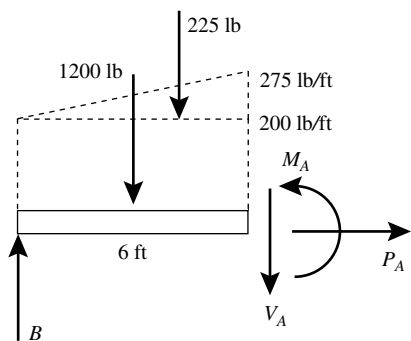
$$\begin{aligned}\sum M_C : -B(8 \text{ ft}) + (1600 \text{ lb})(4 \text{ ft}) \\ + (400 \text{ lb})(2.67 \text{ ft}) - (600 \text{ lb})(1.33 \text{ ft}) &= 0 \\ \Rightarrow B &= 833 \text{ lb}\end{aligned}$$



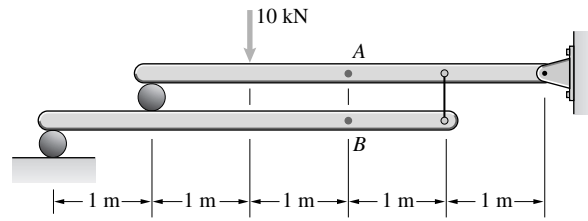
Now examine the section to the left of the cut

$$\begin{aligned}\sum F_x : P_A &= 0 \\ \sum F_y : B - 1200 \text{ lb} - 225 \text{ lb} - V_A &= 0 \\ \sum M_A : -B(6 \text{ ft}) + (1200 \text{ lb})(3 \text{ ft}) \\ + (225 \text{ lb})(2 \text{ ft}) + M_A &= 0\end{aligned}$$

Solving $P_A = 0$, $V_A = -592 \text{ lb}$, $M_A = 950 \text{ ft-lb}$



Problem 10.14 Determine the internal forces and moment at A.



Solution: *The complete structure as a free body:* The sum of the moments about the right end:

$$\sum M = 3(10) - 5R = 0,$$

from which $R = \frac{30}{5} = 6$ kN. The sum of forces in the y-Direction:

$$\sum F_y = R_y + C_y - 10 = 0,$$

from which $C_y = 4$ kN. *The element CA as a free body:* The sum of the moments about C:

$$\sum M_C = -4F_1 + 10(3) - F_2 = 0.$$

The sum of the forces:

$$\sum F_y = C_y + F_2 - 10 + F_1 = 0.$$

Solve the simultaneous equations: $F_1 = 8$ kN, $F_2 = -2$ kN. *Make a cut at A:* Isolate the left end of CA. The sum of the moments about A:

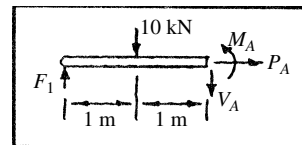
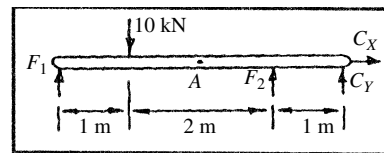
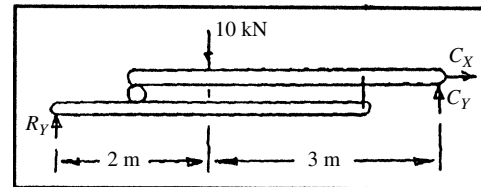
$$\sum M = M_A - 2F_1 + 10 = 0,$$

from which

$$M_A = -10 + 16 = 6 \text{ kN-m}$$

$$V_A = 8 - 10 = -2 \text{ kN},$$

$$P_A = 0$$



Problem 10.15 Determine the internal forces and moment at point B in Problem 10.14.

Solution: Use the solutions to Problem 10.14. *Make a cut at point B:* Isolate the left part. The sum of the moments about B:

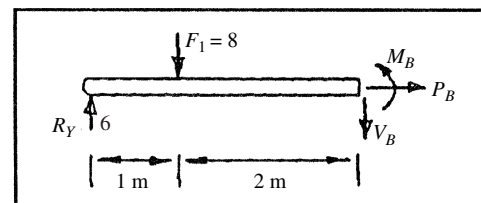
$$\sum M_B = M_B + 2F_1 - 3R_y = 0,$$

from which

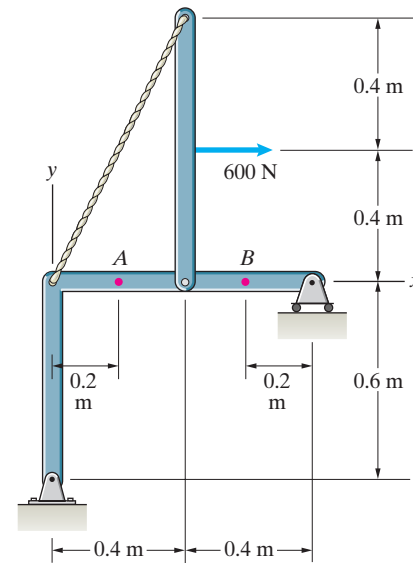
$$M_B = -16 + 18 = 2 \text{ kN-m}$$

$$V_B = R_y - F_1 = 6 - 8 = -2 \text{ kN}$$

$$P_B = 0$$



Problem 10.16 Determine the internal forces and moment at A.

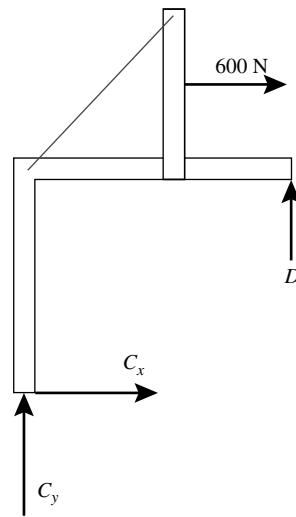


Solution: Use the entire structure to find the reactions

$$\sum F_x : C_x + 600 \text{ N} = 0 \Rightarrow C_x = -600$$

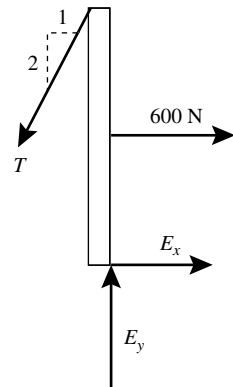
$$\sum M_C : -(600 \text{ N})(1.0 \text{ m}) + D(0.8 \text{ m}) = 0 \Rightarrow D = 750 \text{ N}$$

$$\sum F_y : C_y + D = 0 \Rightarrow C_y = -750 \text{ N}$$



Next examine the vertical bar to find the tension in the cable

$$\sum M_E : -(600 \text{ N})(0.4 \text{ m}) + \frac{1}{\sqrt{5}}T(0.8 \text{ m}) = 0 \Rightarrow T = 671 \text{ N}$$



Finally cut at A and look at the left section

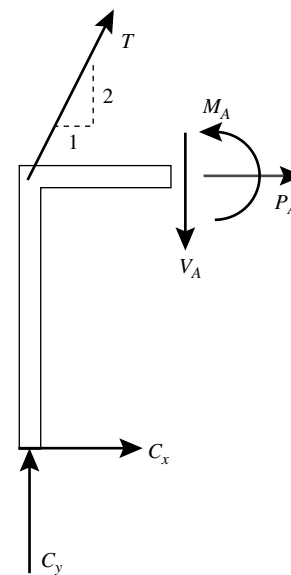
$$\sum F_x : C_x + \frac{1}{\sqrt{5}}T + P_A = 0$$

$$\sum F_y : C_y + \frac{2}{\sqrt{5}}T - V_A = 0$$

$$\sum M_A : C_x(0.6 \text{ m}) - C_y(0.2 \text{ m}) - \frac{2}{\sqrt{5}}T(0.2 \text{ m}) + M_A = 0$$

Solving we have

$$P_A = 300 \text{ N}, \quad V_A = -150 \text{ N}, \quad M_A = 330 \text{ N-m}$$



Problem 10.17 Determine the internal forces and moment at point B of the frame in Problem 10.16.

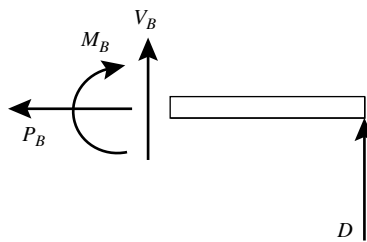
Solution: Use the section to the right of the cut at B

$$\sum F_x : -P_B = 0$$

$$\sum F_y : V_B + D = 0$$

$$\sum M_B : -M_B + D(0.2 \text{ m}) = 0$$

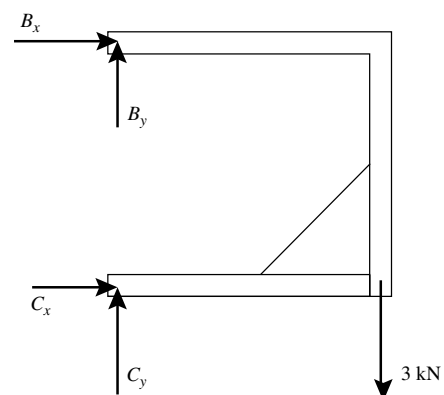
Solving $P_B = 0, \quad V_B = -750 \text{ N}, \quad M_B = 150 \text{ N}\cdot\text{m}$



Problem 10.18 The tension in the rope is 10 kN. Determine the internal forces and moment at point A .

Solution: Use the whole structure first

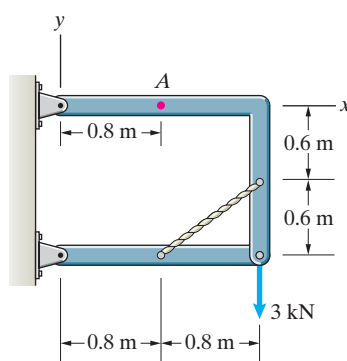
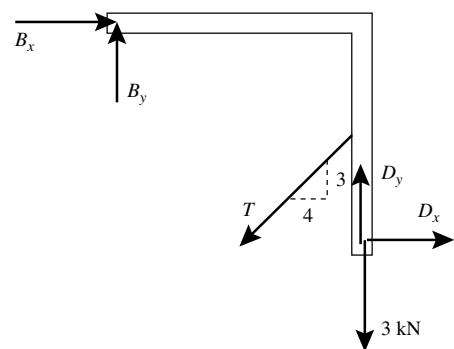
$$\sum M_C : -B_x(1.2 \text{ m}) - (3 \text{ kN})(1.6 \text{ m}) = 0 \Rightarrow B_x = -4 \text{ kN}$$



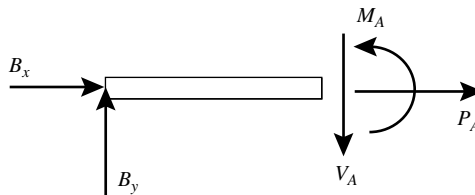
Now examine the bent bar ($T = 10 \text{ kN}$)

$$\sum M_D : -B_x(1.2 \text{ m}) - B_y(1.6 \text{ m}) + \frac{4}{5}(T)(0.6 \text{ m}) = 0$$

$$\Rightarrow B_y = 6 \text{ kN}$$



Finally cut the bar at A and examine the left section



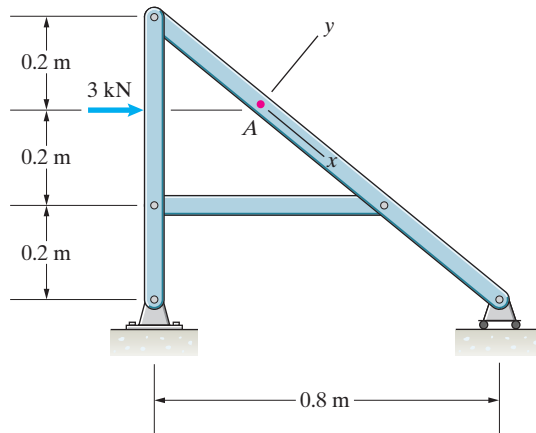
$$\sum F_x : B_x + P_A = 0$$

$$\sum F_y : B_y - V_A = 0$$

$$\sum M_A : -B_y(0.8 \text{ m}) + M_A = 0$$

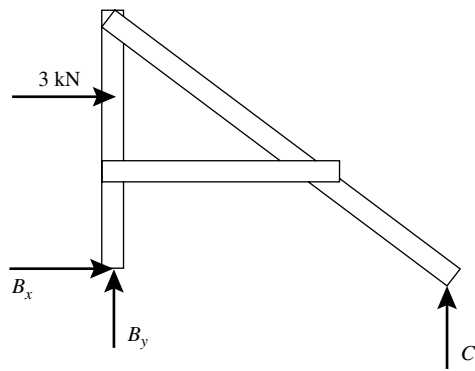
$$\Rightarrow \begin{aligned} P_A &= 4 \text{ kN} \\ V_A &= 6 \text{ kN} \\ M_A &= 4.8 \text{ kN}\cdot\text{m} \end{aligned}$$

Problem 10.19 Determine the internal forces and moment at point A of the frame.



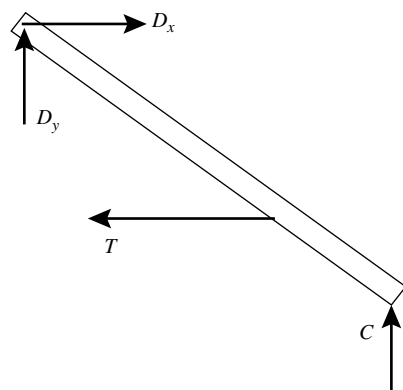
Solution: Use the whole structure first

$$\sum M_B : -(3 \text{ kN})(0.4 \text{ m}) + C(0.8 \text{ m}) = 0 \Rightarrow C = 1.5 \text{ kN}$$



Next examine the slanted bar and take advantage of the 2-force member

$$\sum M_D : C(0.8 \text{ m}) - T(0.4 \text{ m}) = 0 \Rightarrow T = 3 \text{ kN}$$



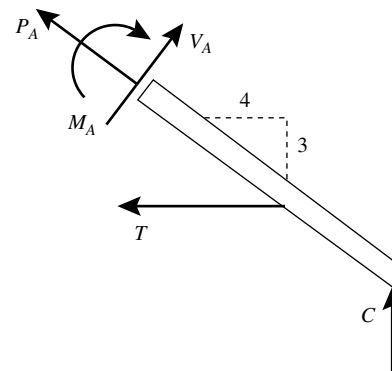
Now cut the bar at A and look at the lower right section

$$\sum F_{\nearrow} : V_A - \frac{3}{5}T + \frac{4}{5}C = 0$$

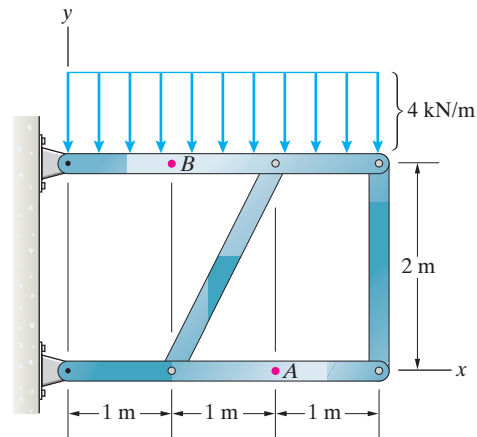
$$\sum F_{\searrow} : P_A + \frac{4}{5}T + \frac{3}{5}C = 0$$

$$\sum M_A : -T(0.2 \text{ m}) + C\left(\frac{2}{3}0.8 \text{ m}\right) - M_A = 0$$

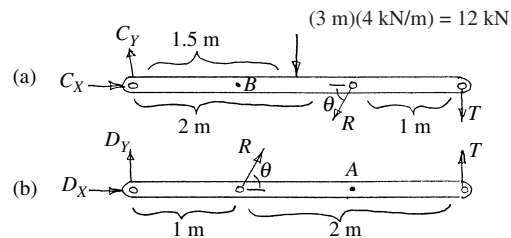
Solving $P_A = -3.3 \text{ kN}$, $V_A = 0.6 \text{ kN}$, $M_A = 0.2 \text{ kN-m}$



Problem 10.20 Determine the internal forces and moment at A.



Solution: The free-body diagrams of the horizontal members are



The angle $\theta = \arctan(2/1) = 63.4^\circ$.

From free-body diagram (a),

$$\sum F_x = C_x - R \cos \theta = 0,$$

$$\sum F_y = C_y - 12 - R \sin \theta - T = 0,$$

$$\sum M_{(\text{pt. } C)} = -(1.5)(12) - 2R \sin \theta - 3T = 0,$$

and from free-body diagram (b),

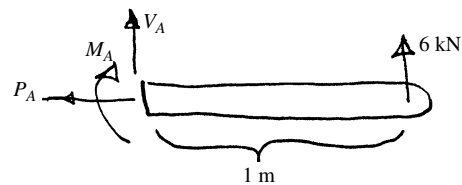
$$\sum F_x = D_x + R \cos \theta = 0,$$

$$\sum F_y = D_y + R \sin \theta + T = 0,$$

$$\sum M_{(\text{pt. } D)} = (1)R \sin \theta + 3T = 0.$$

Solving, we obtain $C_x = -9 \text{ kN}$, $C_y = 0$, and $T = 6 \text{ kN}$.

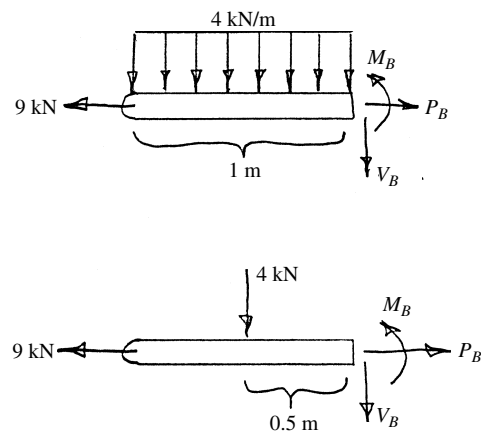
Cutting member (b) at A,



we see that $P_A = 0$, $V_A = -6 \text{ kN}$, $M_A = (1)(6) = 6 \text{ kN-m}$.

Problem 10.21 Determine the internal forces and moment at point B of the frame in Problem 10.20.

Solution: See the solution of Problem 10.20. Cutting member (a) at B and including the distributed load acting on the part of the member to the left of B,



we see that $P_B = 9 \text{ kN}$, $V_B = -4 \text{ kN}$, $M_B = -(0.5)(4) = -2 \text{ kN-m}$.

Problem 10.22 Determine the shear force and bending moment as functions of x .

Strategy: Cut the beam at an arbitrary position x and draw the free-body diagram of the part of the beam to the left of the plane.

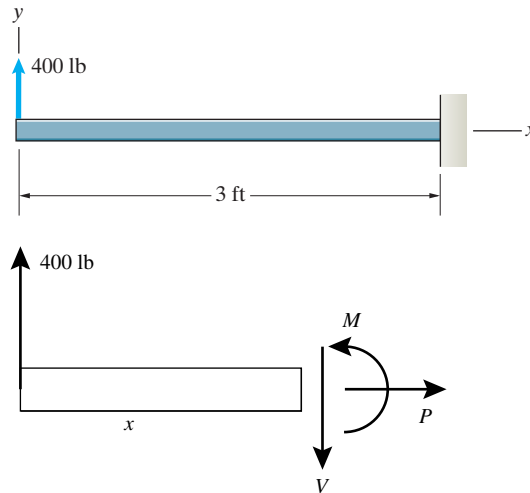
Solution: Cut the beam at arbitrary position x and look at section to the left of the cut

$$\sum F_y : 400 \text{ lb} - V = 0$$

$$\sum M_{\text{cut}} : -(400 \text{ lb})x + M = 0$$

Solving we have

$$\begin{aligned} V &= 400 \text{ lb} \\ M &= (400 \text{ lb})x \end{aligned}$$

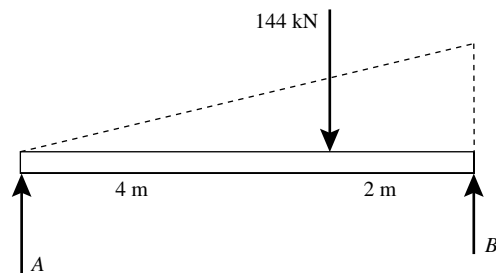


Problem 10.23 (a) Determine the shear force and bending moment as functions of x .

(b) Draw the shear force and bending moment diagrams.

Solution: First determine the reactions

$$\sum M_B : -A(6 \text{ m}) + (144 \text{ kN})(2 \text{ m}) = 0 \Rightarrow A = 48 \text{ kN}$$

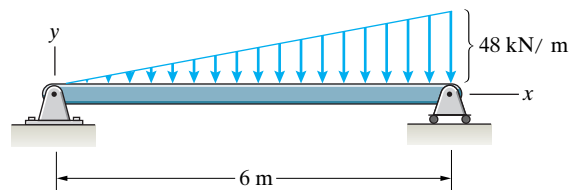
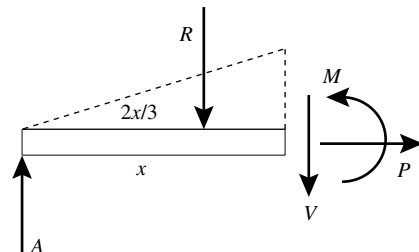


Now cut the beam at arbitrary x and examine the left section.

$$R = \frac{1}{2}x \left(\frac{x}{6 \text{ m}} 48 \frac{\text{kN}}{\text{m}} \right) = \frac{4x^2}{\text{m}^2} \text{ kN}$$

$$\sum F_y : A - R - V = 0$$

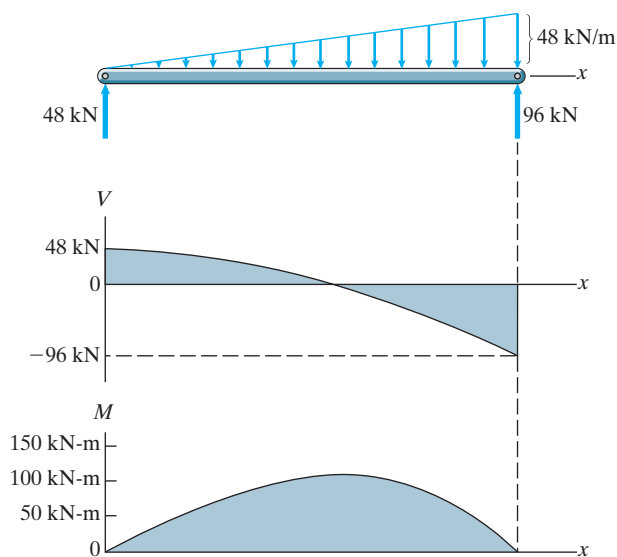
$$\sum M_{\text{cut}} : -Ax + R \left(\frac{x}{3} \right) + M = 0$$



(a) Solving we find

$$V = (48 - 4x^2) \text{ kN}, \quad M = \frac{4}{3}(36x - x^3) \text{ kN-m}$$

(b) The shear and moment diagrams



Problem 10.24 (a) Determine the shear force and bending moment as functions of x .

(b) Show that the equations for V and M as functions of x satisfy the equation $V = dM/dx$.

Strategy: For part (a), cut the beam at an arbitrary position x and draw the free-body diagram of the part of the beam to the right of the plane.

Solution: Cut the beam at arbitrary x and examine the section to the right

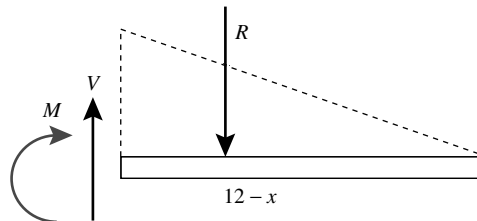
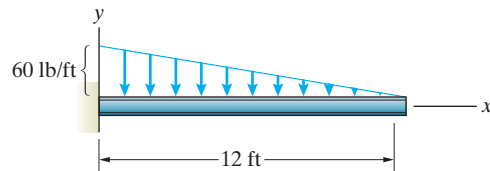
$$R = \frac{1}{2}(12-x) \left(\frac{12-x}{12} \right) (60) = \frac{5}{2}(12-x)^2$$

$$\sum F_y : V - R = 0$$

$$\sum M_{\text{cut}} : -M - R \left(\frac{12-x}{3} \right) = 0$$

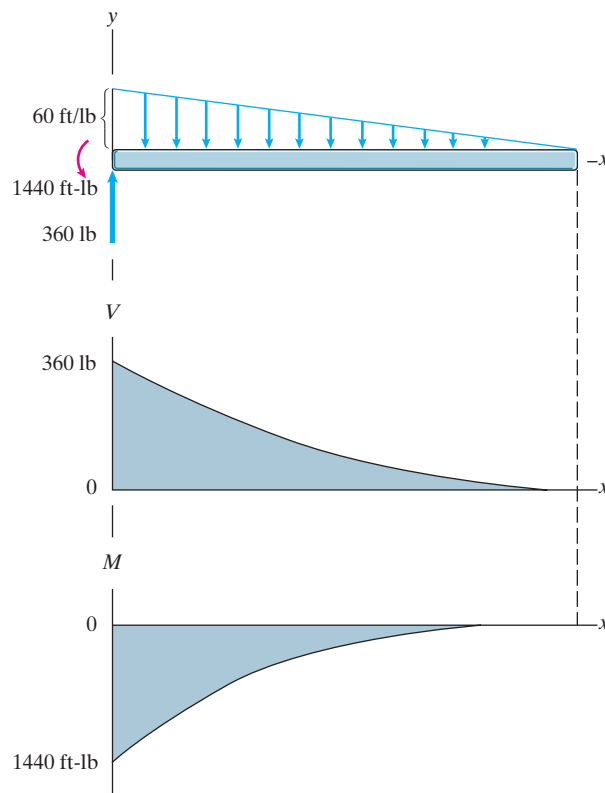
(a) Solving $V = \frac{5}{2}(x-12)^2 \text{ lb}, M = \frac{5}{6}(x-12)^3 \text{ ft-lb}$

(b) $\frac{dM}{dx} = \frac{5}{2}(x-12)^2 = V$

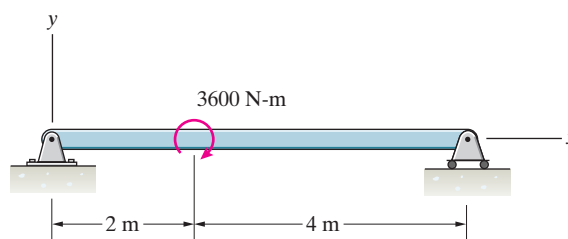


Problem 10.25 Draw the shear force and bending moment diagrams for the beam in Problem 10.24.

Solution: The diagrams



Problem 10.26 Determine the shear force and bending moment as functions of x for $0 < x < 2$ m.



Solution: Examine the whole beam first

$$\sum M_B : -A(6 \text{ m}) - 3600 \text{ N-m} = 0 \Rightarrow A = -600 \text{ N}$$

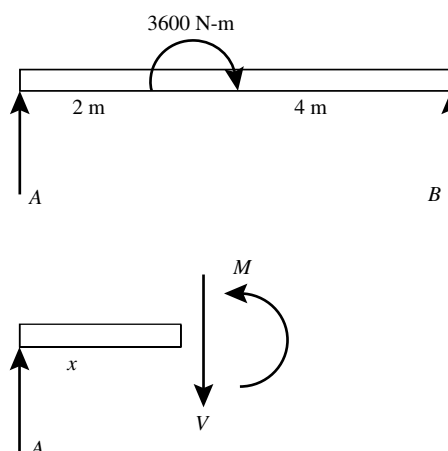
$$\sum F_y : A + B = 0 \Rightarrow B = 600 \text{ N}$$

Now cut at arbitrary $x < 2$ m and examine the left cut

$$\sum F_y : A - V = 0$$

$$\sum M_{\text{cut}} : -Ax + M = 0$$

Solving $V = -600 \text{ N}$, $M = -(600 \text{ N})x$ for $0 < x < 2$ m



Problem 10.27 In Active Example 10.3, suppose that the 40 kN/m distributed load extends all the way across the beam from A to C. Draw a sketch of the beam with its new loading. Determine the shear force V and bending moment M for the beam as functions of x for $2 < x < 4$ m.

Solution: Cutting the beam at an arbitrary position x in the range $2 \text{ m} < x < 4 \text{ m}$, we obtain a free-body diagram of the part of the beam to the right of x . We represent the distributed load by an equivalent force.

From the equilibrium equations

$$\sum F_y : V - (40 \text{ kN/m})(4 \text{ m} - x) + 60 \text{ kN} = 0,$$

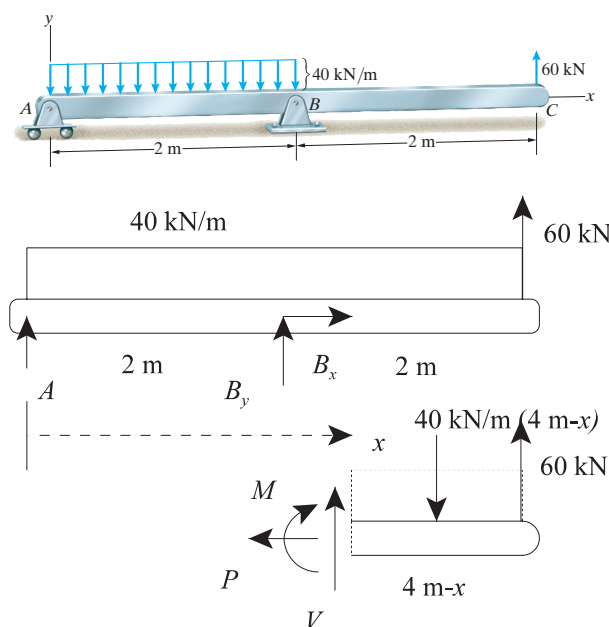
$$\sum M_{\text{left end}} : -M + (60 \text{ kN})(4 \text{ m} - x)$$

$$- [(40 \text{ kN/m})(4 \text{ m} - x)]\left[\frac{1}{2}(4 \text{ m} - x)\right] = 0.$$

We obtain

$$V(x) = 100 \text{ kN} - (40 \text{ kN/m})x,$$

$$M(x) = -80 \text{ kN-m} + (100 \text{ kN})x - (20 \text{ kN/m})x^2.$$



Problem 10.28

- Determine the internal forces and moment as functions of x .
- Determine the shear force and bending moment diagrams.

Solution: The reactions at the left end: The area under the load distribution is

$$F = \left(\frac{1}{2}\right)(6)(100) = 300 \text{ lb.}$$

The centroidal distance is

$$d = 6 + \left(\frac{2}{3}\right)6 = 10 \text{ ft.}$$

The sum of the moments about the left end:

$$\sum M = M^A - dF = 0,$$

from which $M^A = 10(300) = 3000 \text{ ft lb}$. The shear and moment as a function of x : Divide the beam into two intervals: $0 \leq x \leq 6$, and $6 < x \leq 12$. *Interval 1:* The shear as a function of x : $V_1(x) = F = 300 \text{ lb}$. The moment: $M_1(x) = Fx - M^A$, from which $M_1(x) = 300x - 3000 \text{ ft lb}$. *Interval 2:* The load curve is a straight line, with intercept -100 lb/ft , and slope $\frac{100}{6}$. The shear diagram over this interval is

$$\begin{aligned} V_2(x) &= F - \int_6^x \left(-100 + \frac{100}{6}x\right) dx \\ &= F - 100 \left[-x + \frac{1}{12}x^2\right]_6^x = 100 \left(x - \frac{x^2}{12}\right) \end{aligned}$$

Check: The load curve is continuous at $x = 6$, hence the shear diagram must be continuous at $x = 6$, (since the integral of a continuous function is also continuous) hence

$$V_1(6) = V_2(6)$$

$$V_1(6) = F = 300 = V_2(6)$$

$$= 100(6) - 100 \left(\frac{36}{12}\right) = 300$$

check. The force due to the distributed load in the interval $6 \leq x \leq 12$ is

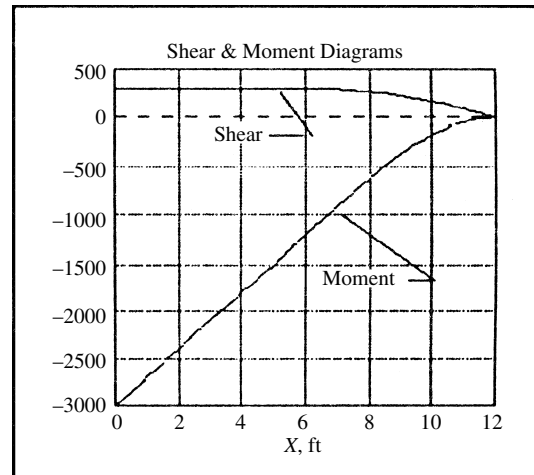
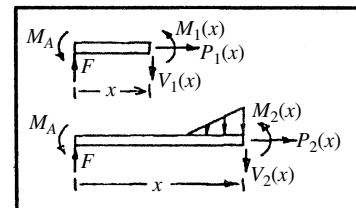
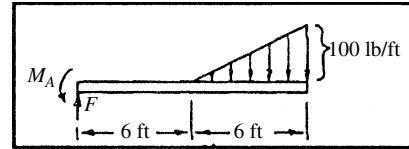
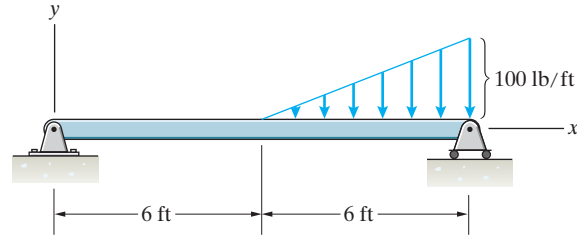
$$F_2(x) = \int_6^x w dx.$$

Integrate and reduce:

$$F_2(x) = 300 - 100x + \frac{25}{3}x^2.$$

The centroid distance from x is

$$d_2 = \frac{(x-6)}{3}.$$



The moment about x is $M_2(x) = Fx - M^A - d_2F_2$. Substitute and reduce:

$$M_2(x) = -2400 + 50x^2 - \left(\frac{25}{9}\right)x^3 \text{ ft lb.}$$

Check: The moment must be zero at $x = 12$. *check.* *Check:* The moment must be continuous at $x = 6$, $M_1(6) = M_2(6)$, from which $M_1(6) = -1200$, and $M_2(6) = -1200$ *check.* The axial forces are zero, $P(x) = 0$

(b) The graph was drawn with **TK Solver Plus**.

Problem 10.29 The loads $F = 200 \text{ N}$ and $C = 800 \text{ Nm}$.

- Determine the internal forces and moment as functions of x .
- Draw the shear force and bending moment diagrams.

Solution: *The reactions at the supports:* The sum of the moments about the left end:

$$\sum M = C + 8B - 16F = 0,$$

from which

$$B = \left(\frac{1}{8}\right)(-C + 16F) = \left(\frac{1}{8}\right)(-800 + 16(200)) = 300 \text{ N}.$$

The sum of the forces: $\sum F_y = R_y + B - F = 0$,

from which $R_y = -100 \text{ N}$. *The intervals as free bodies:* Divide the interval into three parts: $0 \leq x \leq 4$, $4 < x \leq 8$, and $8 < x \leq 16$. *Interval 1:* The shear is $V_1(x) = R_y = -100 \text{ N}$. The moment is $M_1(x) = +R_y x = -100x \text{ N m}$. *Interval 2:* The shear is $V_2(x) = R_y = -100 \text{ N}$. The sum of the moments is

$$\sum M^x = M_2(x) + C - R_y x = 0,$$

from which, $M_2(x) = -100x - 800 \text{ Nm}$. *Interval 3:* The shear is $V_3(x) = R_y + B = -100 + 300 = 200 \text{ N}$. The sum of the moments is

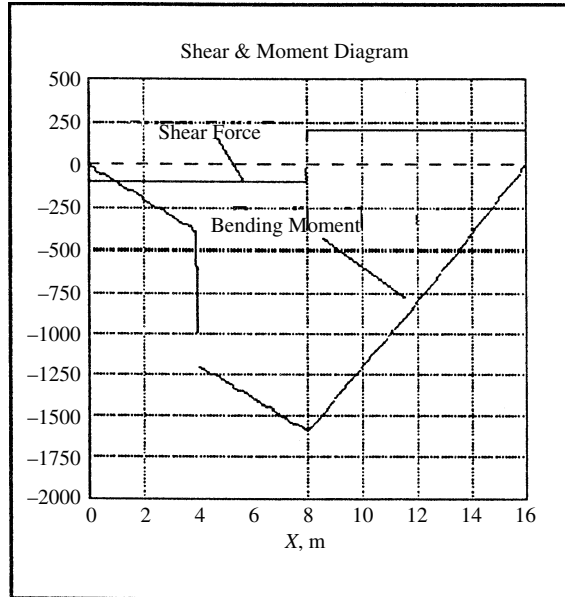
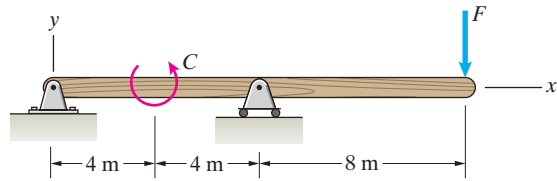
$$\sum M^x = M_3(x) + C - R_y x - B(x - 8) = 0,$$

from which

$$M_3(x) = -800 - 100x + 300x - 2400 = 200x - 3200 \text{ N m}.$$

The axial forces are zero, $P(x) = 0$ in all intervals.

(b) The diagrams are shown.



Problem 10.30 The beam in Problem 10.29 will safely support shear forces and bending moments of magnitudes 2 kN and 6.5 kN-m , respectively. On the basis of this criterion, can it safely be subjected to the loads $F = 1 \text{ kN}$, $C = 1.6 \text{ kN-m}$?

Solution: From the solution to Problem 10.29, the shear and the moments in the intervals are *Interval 1:* $V_1 = R_y$, $M_1(x) = R_y x$, *Interval 2:* $V_2(x) = R_y$, $M_2(x) = R_y x + C$, *Interval 3:* $V_3(x) = R_y + B$, $M_3(x) = (R_y + B)x + C - 8B$. The reactions are

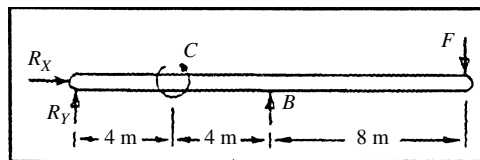
$$B = \left(\frac{1}{8}\right)(16F - C),$$

and $R_y = F - B$.

The maximum shears in each interval have the magnitude rank:

$$|V_1(x)| = |V_2(x)| \leq |V_3(x)|,$$

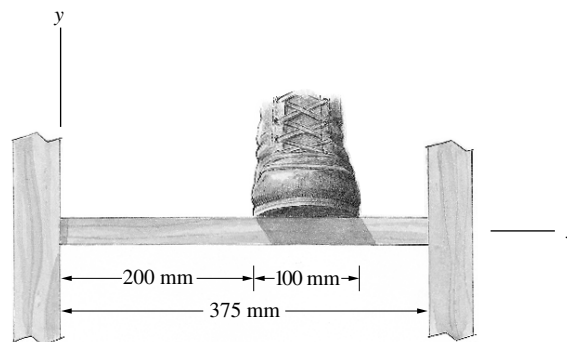
so that the largest shear for a force $F = 1 \text{ kN}$ is $V_3(x) = R_y + B = F - B + B = F = 1 \text{ kN}$, which can be safely supported. The maximum moment magnitudes in each interval have the rank: $|M_1(x)| \leq |M_2(x)| \leq |M_3(x)|$. The maximum moment magnitude occurs in the third interval:



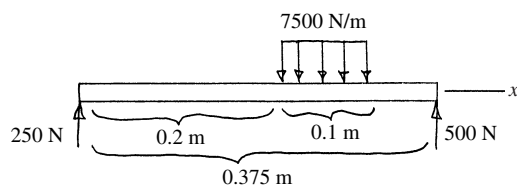
$$\begin{aligned} M_3(x) &= (R_y + B)x + C - 8B = Fx + C - (16F - C) \\ &= Fx - 16F + 2C. \end{aligned}$$

The maximum magnitude occurs at $x = 8$, $|M_3(8)| = 8F = 8 \text{ kN m}$ and it exceeds the safe limit by 2.5 kN m . *NO*

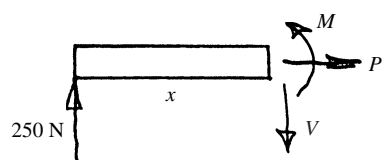
Problem 10.31 Model the ladder rung as a simply supported (pin-supported) beam and assume that the 750-N load exerted by the person's shoe is uniformly distributed. Draw the shear force and bending moment diagrams.



Solution: See the solution of Problem 10.7. The free-body diagram of the rung is

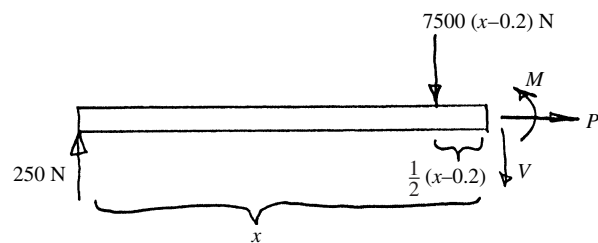
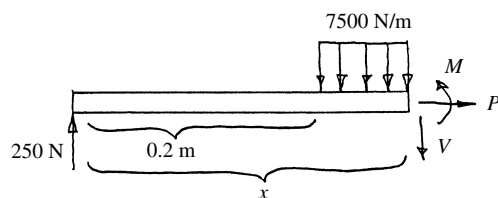


$$0 < x < 0.2 \text{ m}$$



$$V = 250 \text{ N}, \quad M = 250x \text{ N}\cdot\text{m}.$$

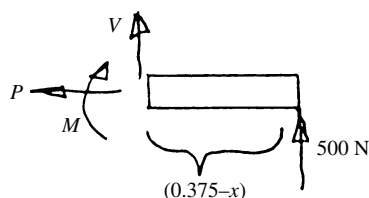
$$0.2 < x < 0.3 \text{ m}$$



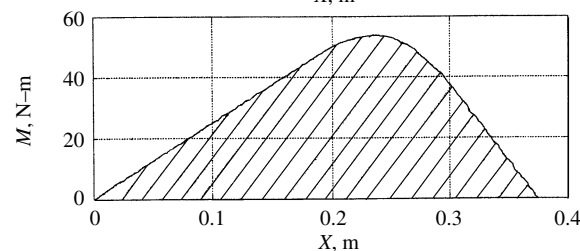
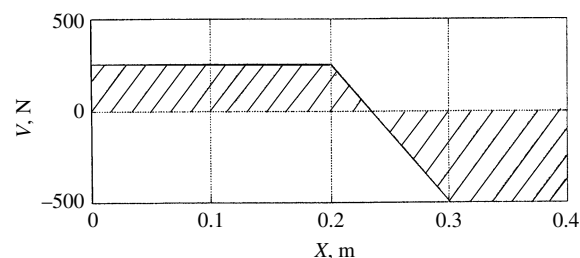
$$V = 250 - 7500(x - 0.2) \text{ N},$$

$$M = 250x - \frac{1}{2}7500(x - 0.2)^2 \text{ N}\cdot\text{m}.$$

$$0.3 < x < 0.375 \text{ m}$$



$$V = -500 \text{ N}, \quad M = 500(0.375 - x) \text{ N}\cdot\text{m}.$$



Problem 10.32 What is the maximum bending moment in the ladder rung in Problem 10.31 and where does it occur?

Solution: See the solution of Problem 10.31. The maximum moment occurs in the interval $0.2 < x < 0.3$ m, in which

$$M = 250x - 3750(x - 0.2)^2 \text{ N}\cdot\text{m}.$$

$$\text{Setting } \frac{dM}{dx} = 250 - 7500(x - 0.2) = 0,$$

we find that the maximum moment occurs at $x = 0.233$ m. Substituting this value into the expression for M gives $M = 54.2$ N·m.

Problem 10.33 Assume that the surface the beam rests on exerts a uniformly distributed load. Draw the shear force and bending moment diagrams.

Solution: The load density is $w = \frac{6}{6} = 1$ kN/m.

The intervals as free bodies: Divide the beam into three intervals:

$$0 \leq x < 2 \text{ (m)},$$

$$2 \leq x < 5 \text{ (m)},$$

and $5 \leq x \leq 6 \text{ (m)}.$

Interval 1: The shear force is

$$V_1(x) = \int_0^x w \, dx = x \text{ kN}.$$

The force to the left is

$$F_1(x) = \int_0^x w \, dx = x \text{ kN}.$$

The centroid distance from x is $\bar{d}_1 = \frac{x}{2}$.

The moment is

$$M_1(x) = F_1(x)\bar{d}_1 = \frac{x^2}{2} \text{ kN}\cdot\text{m}.$$

Interval 2: The shear force is

$$V_2(x) = V_1(x) - 4 = x - 4 \text{ kN}.$$

The moment is

$$M_2(x) = M_1(x) - 4(x - 2) = \frac{x^2}{2} - 4x + 8 \text{ kN}\cdot\text{m}.$$

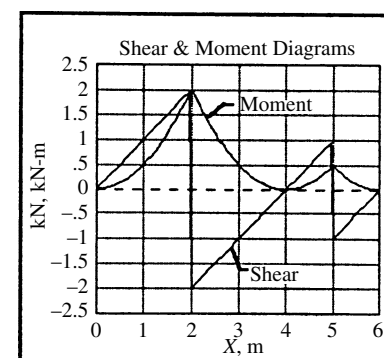
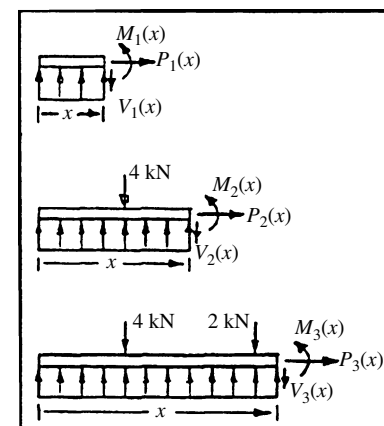
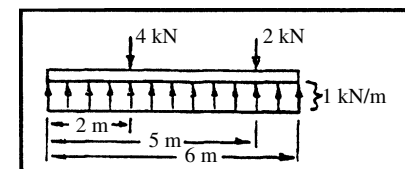
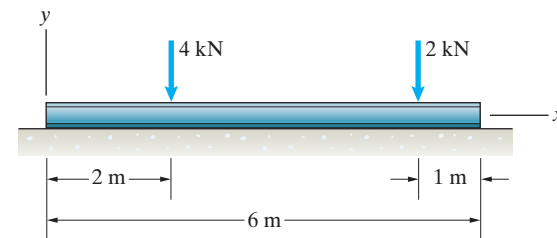
Interval 3: The shear force is

$$V_3(x) = x - 4 - 2 = x - 6 \text{ kN}.$$

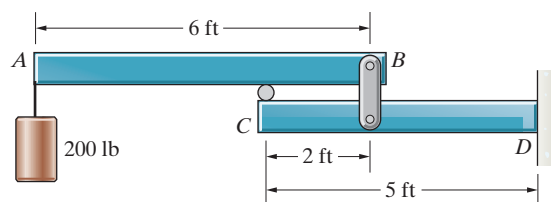
The bending moment is

$$M_3(x) = M_2(x) - 2(x - 5) = \frac{x^2}{2} - 6x + 18 \text{ kN}\cdot\text{m}.$$

The shear and moment diagrams are shown.



Problem 10.34 The homogeneous beams AB and CD weigh 600 lb and 500 lb, respectively. Draw the shear force and bending moment diagrams for beam AB .



Solution: Find reactions

$$\sum M_B : (200 \text{ lb})(6 \text{ ft}) + (600 \text{ lb})(3 \text{ ft}) - C(2 \text{ ft}) = 0$$

$$\sum F_y : -200 \text{ lb} - 600 \text{ lb} + C - B = 0$$

$$\Rightarrow C = 1500 \text{ lb}, B = 700 \text{ lb}$$

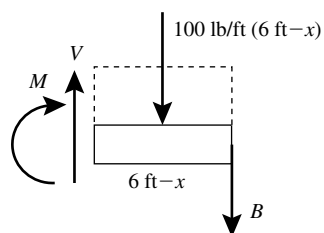
For $0 < x < 4 \text{ ft}$ we have

$$\sum F_y : -200 \text{ lb} - \left(100 \frac{\text{lb}}{\text{ft}}\right)x - V = 0$$

$$\sum M_{\text{cut}} : (200 \text{ lb})x + \left(100 \frac{\text{lb}}{\text{ft}}\right)x \left(\frac{x}{2}\right) + M = 0$$

$$\begin{aligned} V &= -200 \text{ lb} - \left(100 \frac{\text{lb}}{\text{ft}}\right)x \\ M &= -(200 \text{ lb})x - \left(50 \frac{\text{lb}}{\text{ft}}\right)x^2 \end{aligned}$$

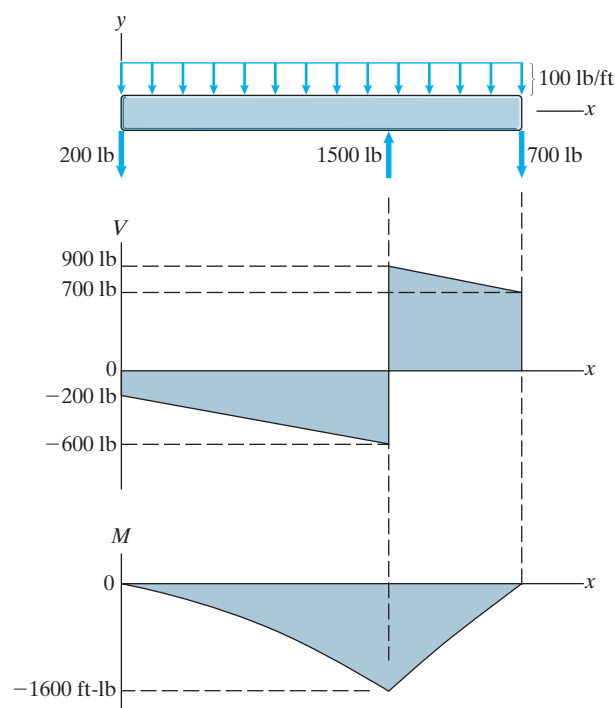
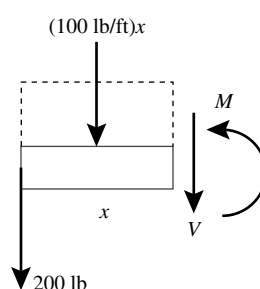
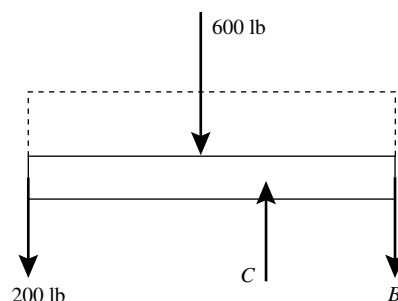
For $4 \text{ ft} < x < 6 \text{ ft}$ we have



$$\sum F_y : V - B - 100 \frac{\text{lb}}{\text{ft}}(6 \text{ ft} - x) = 0$$

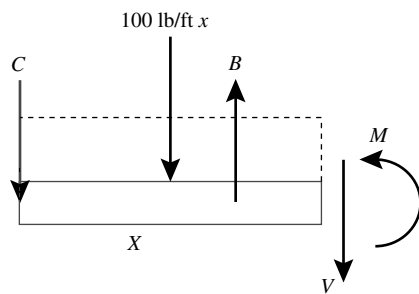
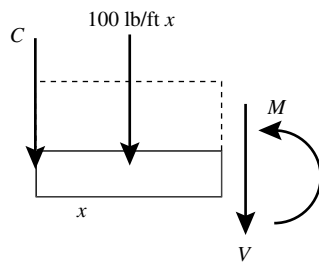
$$\sum M_{\text{cut}} : -M - \left(100 \frac{\text{lb}}{\text{ft}}\right) \frac{(6 \text{ ft} - x)^2}{2} - B(6 \text{ ft} - x) = 0$$

$$\begin{aligned} V &= 700 \text{ lb} - (100 \text{ lb/ft})(x - 6 \text{ ft}), \\ M &= 700 \text{ lb}(x - 6 \text{ ft}) - (50 \text{ lb/ft})(x - 6 \text{ ft})^2 \end{aligned}$$



Problem 10.35 Draw the shear force and bending moment diagrams for beam CD in Problem 10.34.

Solution: Use the reactions from 10.34



For $0 < x < 2$ ft

$$\sum F_y : -C - V - (100 \text{ lb/ft})x = 0$$

$$\sum M_{\text{cut}} : Cx + [(100 \text{ lb/ft})x]x/2 + M = 0$$

$$V = -1500 \text{ lb} - (100 \text{ lb/ft})x$$

$$M = -(1500 \text{ lb})x - (50 \text{ lb/ft})x^2$$

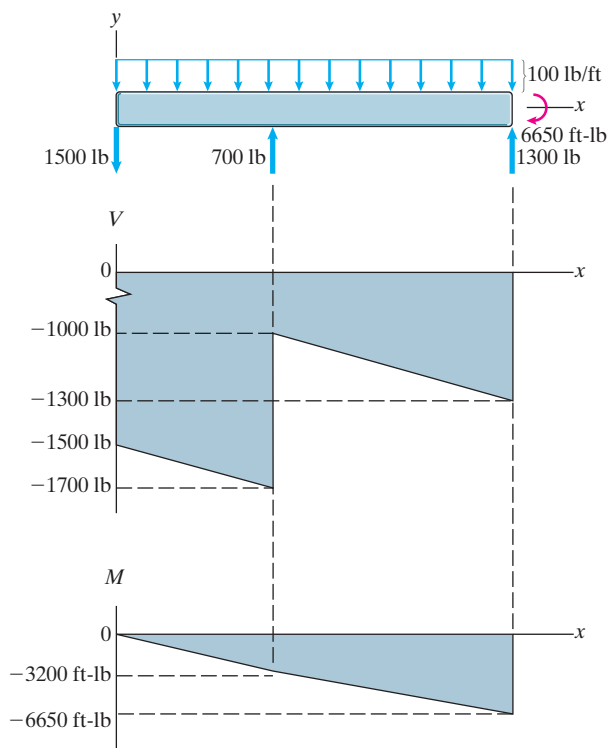
For $2 \text{ ft} < x < 5$ ft

$$\sum F_y : -C + B - (100 \text{ lb/ft})x - V = 0$$

$$\sum M_{\text{cut}} : Cx - B(x - 2 \text{ ft}) + (100 \text{ lb/ft})x^2/2 + M = 0$$

$$V = -800 \text{ lb} - (100 \text{ lb/ft})x$$

$$M = -1400 \text{ ft-lb} - (800 \text{ lb})x - (50 \text{ lb/ft})x^2$$



Problem 10.36 Determine the shear force V and bending moment M for the beam as functions of x for $0 < x < 3$ ft.

Solution: From the free-body diagram of the entire beam (Fig. a), we obtain the equilibrium equations

$$\Sigma F_x : A_x = 0,$$

$$\Sigma F_y : A_y - 900 \text{ lb} + 900 \text{ lb} = 0,$$

$$\Sigma M_A : M_A - (900 \text{ lb})(2 \text{ ft})$$

$$+ (900 \text{ lb})(4 \text{ ft}) = 0.$$

We see that

$$A_x = A_y = 0, M_A = -1800 \text{ ft-lb}.$$

Cutting the beam at an arbitrary position x in the range $0 < x < 3$ ft, we obtain a free-body diagram of the part of the beam to the left of x (Fig. b). The distributed load is replaced by the equivalent force

$$R = \frac{1}{2} \left[\left(\frac{600 \text{ lb/ft}}{3 \text{ ft}} \right) x \right] x = (100 \text{ lb/ft}^2)x^2.$$

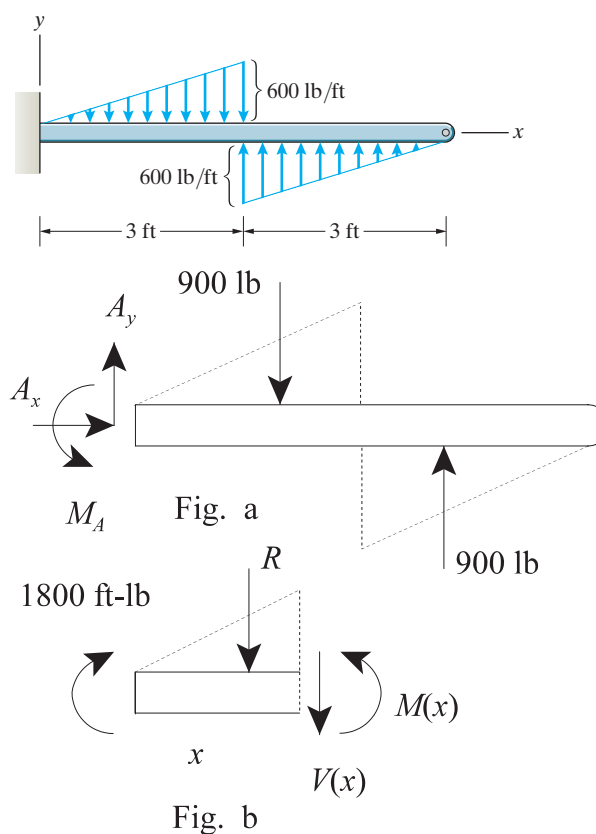
From the equilibrium equations we have

$$\Sigma F_y : -(100 \text{ lb/ft}^2)x^2 - V(x) = 0,$$

$$\Sigma M_{\text{right end}} : -(1800 \text{ lb-ft}) + (100 \text{ lb/ft}^2)x^2(x/3) + M(x) = 0.$$

We obtain

$$V(x) = -(100 \text{ lb/ft}^2)x^2, M(x) = -(33.3 \text{ lb/ft}^2)x^3 + (1800 \text{ lb-ft}).$$



Problem 10.37 Draw the shear force and bending moment diagrams for the beam.

Solution: In the solution to Problem 10.36 we found that in the range $0 < x < 3$ ft, we have

$$V(x) = -(100 \text{ lb/ft}^2)x^2$$

$$M(x) = -(33.3 \text{ lb/ft}^2)x^3 + (1800 \text{ lb-ft}).$$

Cutting the beam at an arbitrary position x in the range $3 \text{ ft} < x < 6$ ft, and isolating the right part (Fig. a), we have the free-body diagram shown. The distributed load is replaced by the equivalent force

$$R = \frac{1}{2} \left[\left(600 \frac{\text{lb}}{\text{ft}} \right) \left(\frac{6 \text{ ft} - x}{3 \text{ ft}} \right) \right] (6 \text{ ft} - x) = (100 \text{ lb/ft}^2)(6 \text{ ft} - x)^2$$

From the equilibrium equations we learn

$$\Sigma F_y : V(x) + (100 \text{ lb/ft}^2)(6 \text{ ft} - x)^2 = 0,$$

$$\Sigma M_{\text{left end}} : -M(x) + [(100 \text{ lb/ft}^2)(6 \text{ ft} - x)^2] \left[\frac{1}{3}(6 \text{ ft} - x) \right] = 0.$$

Thus

$$V(x) = -(100 \text{ lb/ft}^2)(6 \text{ ft} - x)^2$$

$$M(x) = (33.3 \text{ lb/ft}^2)(6 \text{ ft} - x)^3.$$

The resulting diagrams are shown.

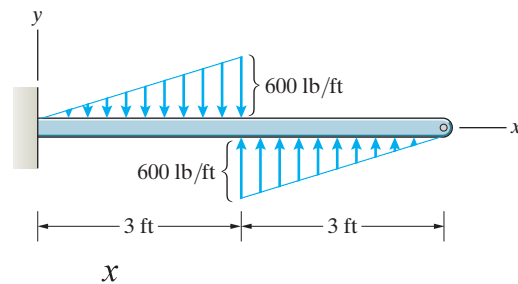
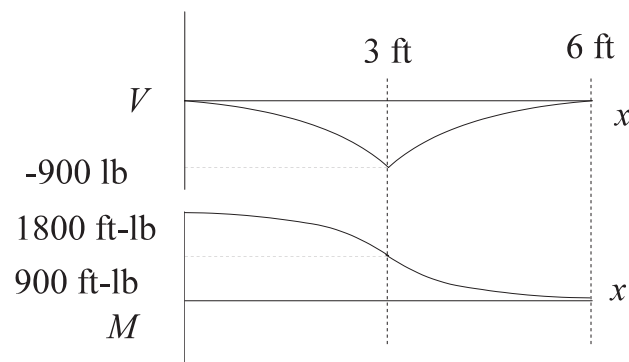
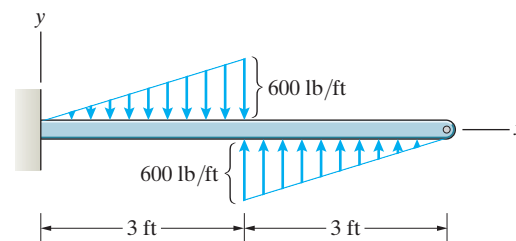
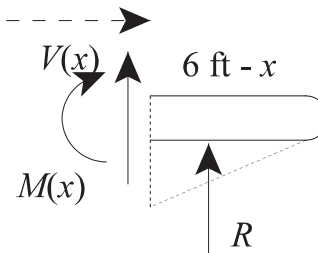


Fig. a



Problem 10.38 In preliminary design studies, the vertical forces on an airplane's wing are modeled as shown. The distributed load models aerodynamic forces and the force exerted by the wing's weight. The 80-kN force at $x = 4.4$ m models the force exerted by the weight of the engine. Draw the shear force and bending moment diagrams for the wing for $0 < x < 4.4$ m.

Solution: From the free-body diagram of the entire wing (Fig. a), we obtain the equilibrium equations

$$\Sigma F_x : A_x = 0,$$

$$\Sigma F_y : A_y + (220 + 325 - 80) \text{ kN} = 0,$$

$$\Sigma M_A : M_A + (220 \text{ kN})(2.2 \text{ m})$$

$$- (80 \text{ kN})(4.4 \text{ m})$$

$$+ (325 \text{ kN})(17.4 - 8.67) \text{ m} = 0.$$

Solving yields

$$A_x = 0, A_y = -465 \text{ kN}, M_A = -2970 \text{ kN-m}.$$

Cutting the wing at an arbitrary position x in the range $0 < x < 4$ m, and representing the distributed load by an equivalent force (Fig. b), the equilibrium equations are

$$\Sigma F_y : -465 \text{ kN} + (50 \text{ kN/m})x - V(x) = 0,$$

$$\Sigma M_{\text{right end}} : -2970 \text{ kN-m} + (465 \text{ kN})x$$

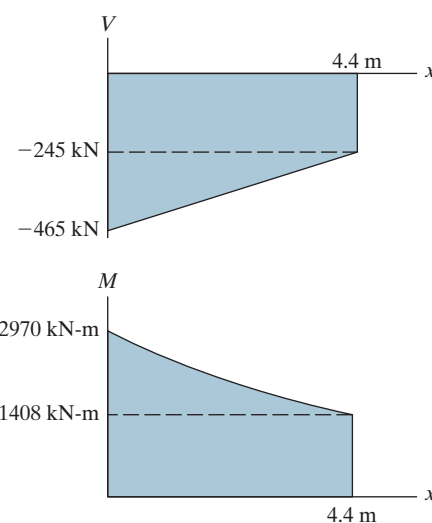
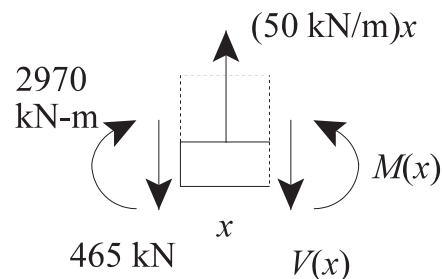
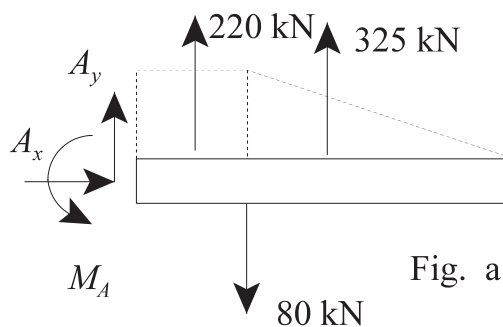
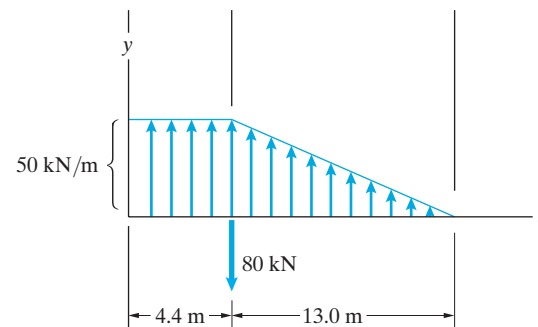
$$- [(50 \text{ kN/m})x](x/2) + M(x) = 0.$$

Therefore

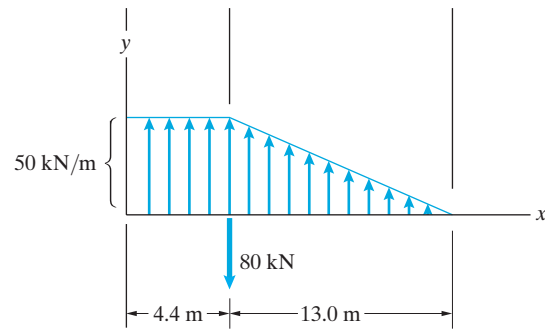
$$V(x) = (50 \text{ kN/m})x - 465 \text{ kN},$$

$$M(x) = (25 \text{ kN/m})x^2 - (465 \text{ kN})x + 2970 \text{ kN-m}.$$

The resulting diagrams are shown.



Problem 10.39 Draw the shear force and bending moment diagram for the entire wing.



Solution: The shear force and bending moment diagrams for $0 < x < 4.4$ m were obtained in the solution to Problem 10.38. Cut the wing at an arbitrary position x in the range $4.4 \text{ m} < x < 17.4 \text{ m}$ and isolate the right part of the beam (Fig. a). The distributed loading is represented by the equivalent force

$$R = \frac{1}{2} \left[\left(\frac{17.4 \text{ m} - x}{13.0 \text{ m}} \right) \left(50 \frac{\text{kN}}{\text{m}} \right) \right] (17.4 \text{ m} - x)$$

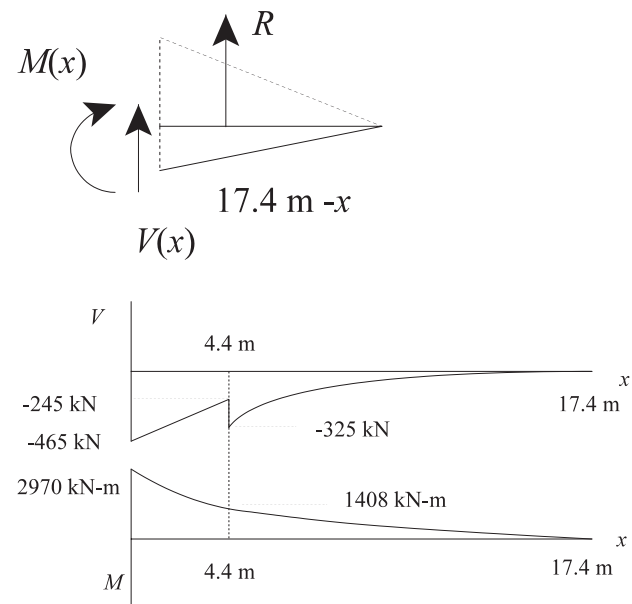
$$= \left(\frac{25 \text{ kN}}{13 \text{ m}^2} \right) (17.4 \text{ m} - x)^2$$

Using the equilibrium equations we find

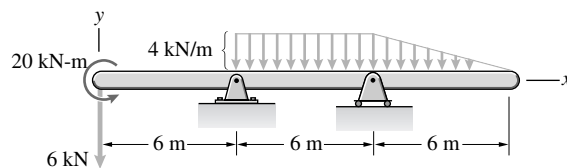
$$V(x) = - \left(\frac{25 \text{ kN}}{13 \text{ m}^2} \right) (17.4 \text{ m} - x)^2,$$

$$M(x) = R \left[\frac{1}{3} (17.4 \text{ m} - x) \right] = \left(\frac{25 \text{ kN}}{39 \text{ m}^2} \right) (17.4 \text{ m} - x)^3$$

The diagrams for the entire beam are shown.



Problem 10.40* Draw the shear force and bending moment diagrams.



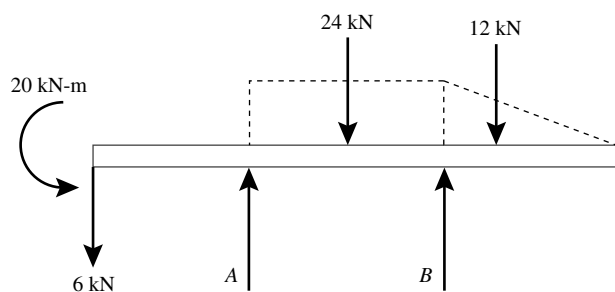
Solution: Start with the reactions

$$\sum M_A : (6 \text{ kN})(6 \text{ m}) + 20 \text{ kN-m}$$

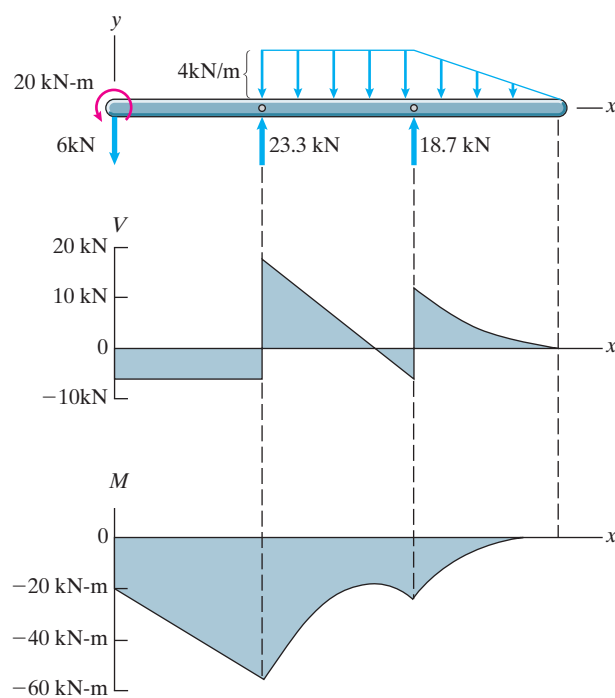
$$- (24 \text{ kN})(3 \text{ m}) - (12 \text{ kN})(8 \text{ m}) + B(6 \text{ m}) = 0$$

$$\sum F_y : A + B - 6 \text{ kN} - 24 \text{ kN} - 12 \text{ kN} = 0$$

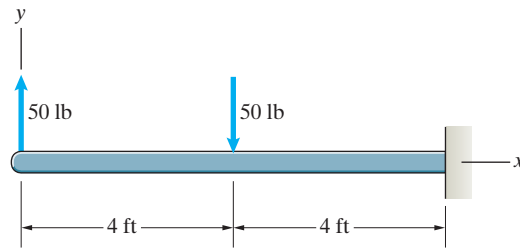
Thus $A = 23.3 \text{ kN}$, $B = 18.67 \text{ kN}$



The complete diagrams:



Problem 10.41 Draw the shear force and bending moment diagrams.



Solution: In the first region $0 < x < 4$ ft

$$w_1 = 0$$

$$V_1 = 50 \text{ lb}$$

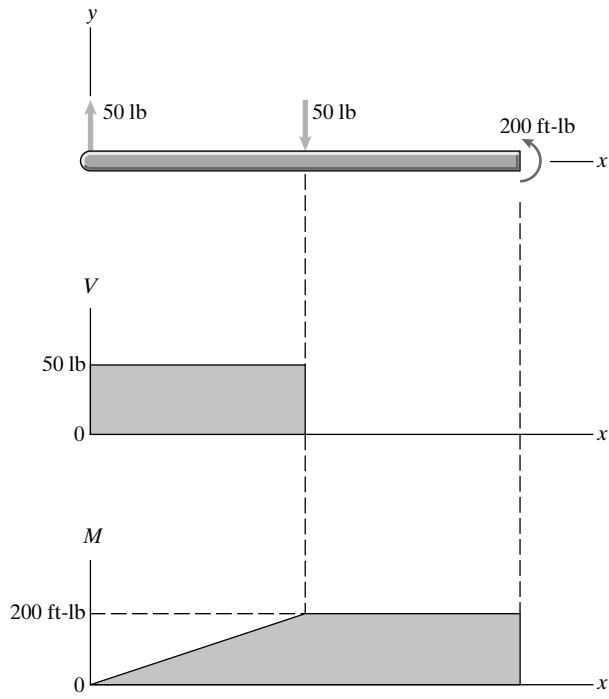
$$M_1 = (50 \text{ lb})x$$

In the second region $4 \text{ ft} < x < 8 \text{ ft}$

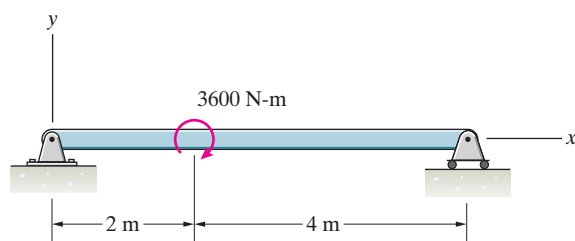
$$w_2 = 0$$

$$V_2 = 0$$

$$M_2 = 200 \text{ ft-lb}$$



Problem 10.42 Draw the shear force and bending moment diagrams.



Solution: We must first find the reactions

$$\sum M_B : -A(6 \text{ m}) - 3600 \text{ N-m} = 0$$

$$\sum F_y : A + B = 0$$

$$A = -600 \text{ N}, B = 600 \text{ N}$$

In the first region $0 < x < 2 \text{ m}$

$$w_1 = 0$$

$$V_1 = A = -600 \text{ N}$$

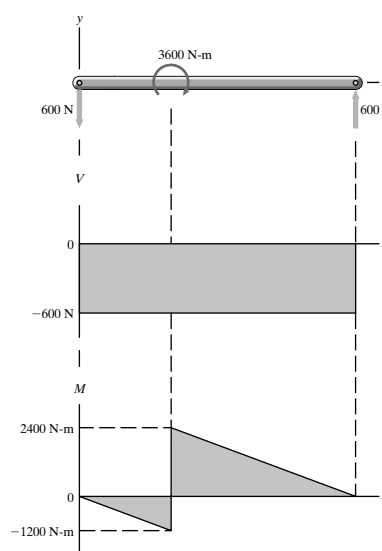
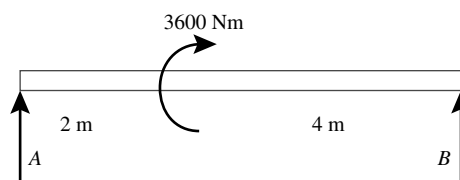
$$M_1 = -(600 \text{ N})x$$

In the second region $2 \text{ m} < x < 6 \text{ m}$

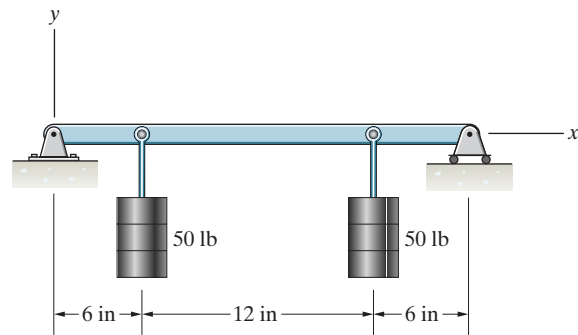
$$w_2 = 0$$

$$V_2 = -B = -600 \text{ N}$$

$$M_2 = -(600 \text{ N})x + 3600 \text{ N-m}$$



Problem 10.43 This arrangement is used to subject a segment of a beam to a uniform bending moment. Draw the shear force and bending moment diagrams.



Solution: We first find the reactions

$$\sum M_B : -A(24 \text{ in}) + (50 \text{ lb})(6 \text{ in}) + (50 \text{ lb})(18 \text{ in}) = 0$$

$$\sum F_y : A + B - 50 \text{ lb} - 50 \text{ lb} = 0$$

$$A = B = 50 \text{ lb}$$

In the first regions $0 < x < 6 \text{ in}$

$$w_1 = 0$$

$$V_1 = A = 50 \text{ lb}$$

$$M_1 = (50 \text{ lb})x$$

In the second region $6 \text{ in} < x < 18 \text{ in}$

$$w_2 = 0$$

$$V_2 = 0$$

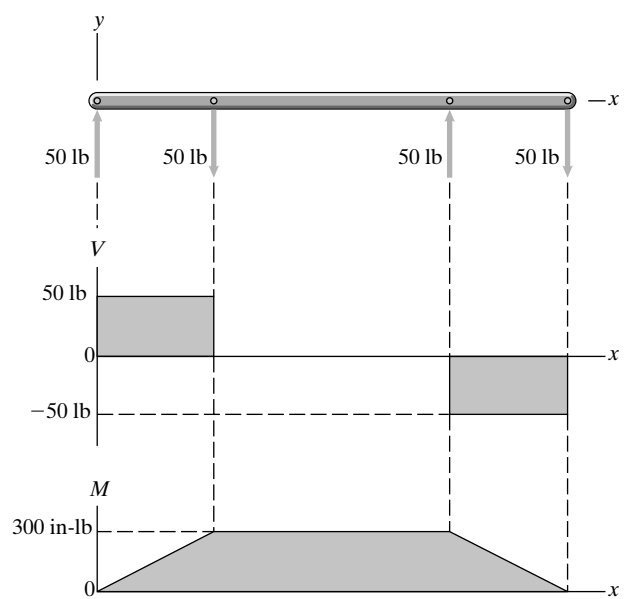
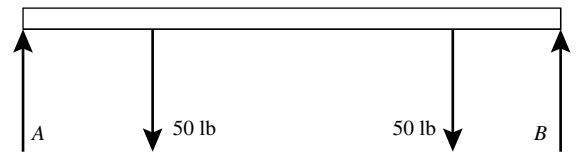
$$M_2 = 300 \text{ in-lb}$$

In the last region $18 \text{ in} < x < 24 \text{ in}$

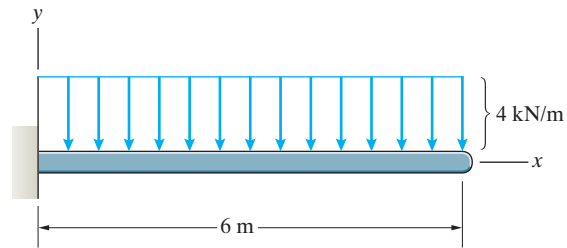
$$w_3 = 0$$

$$V_3 = -B = -50 \text{ lb}$$

$$M_3 = -(50 \text{ lb})x + 1200 \text{ in-lb}$$



Problem 10.44 Use the procedure described in Example 10.5 to draw the shear force and bending moment diagrams for the beam.



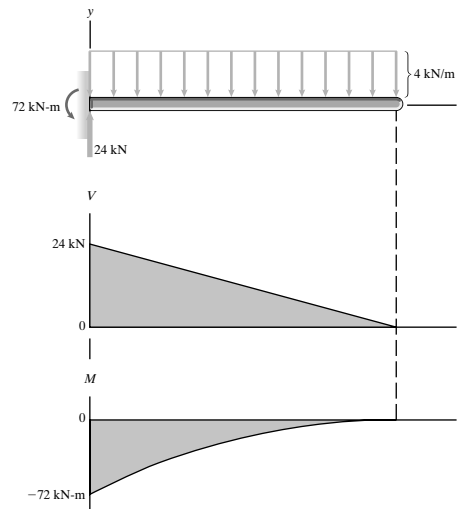
Solution: We can use the boundary conditions at the right end instead of calculating the reactions at the left end

$$w = 4 \frac{\text{kN}}{\text{m}}$$

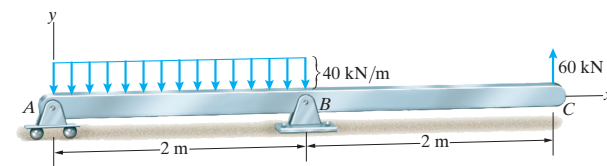
$$V = -\left(4 \frac{\text{kN}}{\text{m}}\right)x + 24 \text{ kN}$$

$$M = -\left(2 \frac{\text{kN}}{\text{m}}\right)x^2 + (24 \text{ kN})x - 72 \text{ kN}\cdot\text{m}$$

The plots



Problem 10.45 In Active Example 10.4, suppose that the 40 kN/m distributed load extends all the way across the beam from A to C. Draw a sketch of the beam with its new loading. Draw the shear force diagram for the beam.



Solution: The free-body diagram with the reactions already solved is shown.

Think of the beginning just to the *left* of the beam, with the initial value of the shear force equal to zero. The upward 60-kN reaction at A causes an increase in the shear force of 60-kN magnitude. Between A and B, the distributed load on the beam is constant—the diagram is a straight line. The change in V between A and B can be determined from Eq. (10.5),

$$V_B - V_A = -(2 \text{ m})(40 \text{ kN/m})$$

$$= -80 \text{ kN}.$$

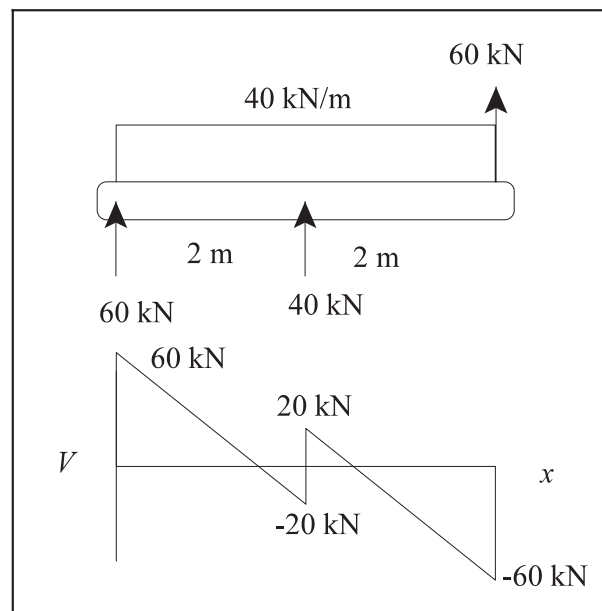
Therefore V decreases linearly from 60 kN at A to 60 kN – 80 kN = –20 kN at B.

The upward 40-kN reaction at B causes an increase in the shear force of 40-kN magnitude, so at B the shear force increases from –20 kN to 20 kN.

Between B and C, the distributed load on the beam is constant, so the shear diagram between B and C is a straight line. The change in V between B and C can be determined from Eq. (10.5).

$$V_C - V_B = -(2 \text{ m})(40 \text{ kN/m}) = -80 \text{ kN}.$$

Therefore V decreases linearly from 20 kN at B to 20 kN – 80 kN = –60 kN at C.



Problem 10.46 Draw the shear force and bending moment diagrams.

Solution: Find the reactions first

$$\sum M_A : -300 \text{ lb}(10 \text{ ft}) + B(12 \text{ ft}) = 0$$

$$\sum F_y : A + B - 300 \text{ lb} = 0$$

$$A = 50 \text{ lb}, B = 250 \text{ lb}$$

For $0 < x < 6 \text{ ft}$

$$w = 0$$

$$\frac{dV}{dx} = -w = 0 \Rightarrow V = 50 \text{ lb}$$

$$\frac{dM}{dx} = V = 50 \text{ lb} \Rightarrow M = (50 \text{ lb})x$$

For $6 \text{ ft} < x < 12 \text{ ft}$

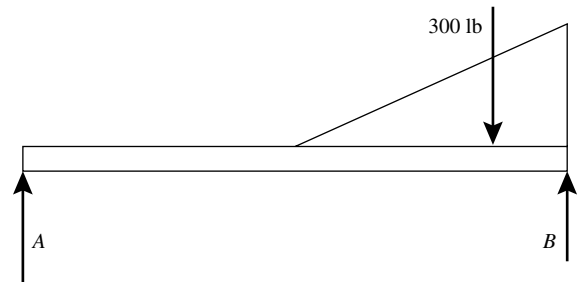
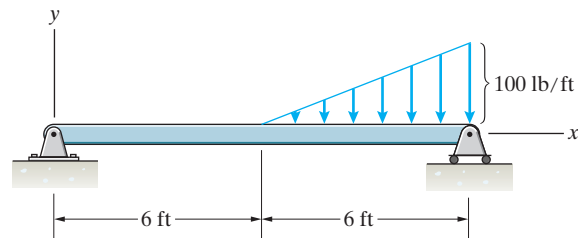
$$w = \left(\frac{100 \text{ lb}}{6 \text{ ft}} \right) (x - 6 \text{ ft}) = \left(\frac{50 \text{ lb}}{3 \text{ ft}} \right) (x - 6 \text{ ft})$$

$$V = - \left(\frac{50 \text{ lb}}{3 \text{ ft}} \right) \frac{(x - 6 \text{ ft})^2}{2} + 50 \text{ lb}$$

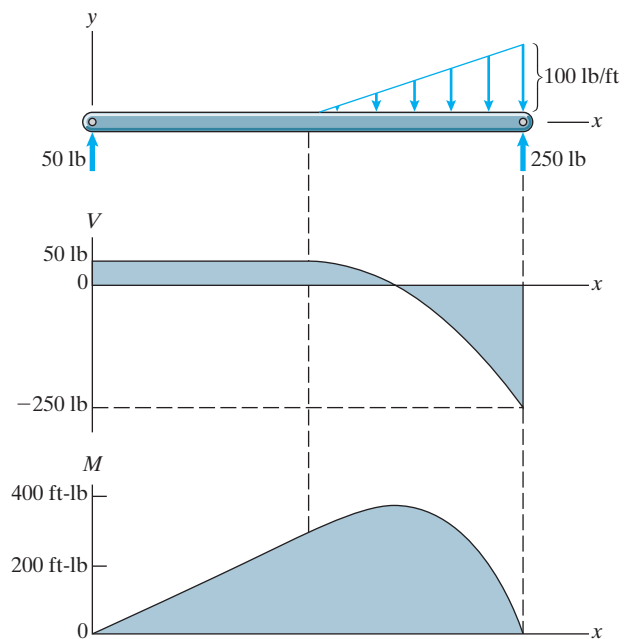
$$= - \left(\frac{25 \text{ lb}}{3 \text{ ft}} \right) (x - 6 \text{ ft})^2 + 50 \text{ lb}$$

$$M = - \left(\frac{25 \text{ lb}}{3 \text{ ft}} \right) \frac{(x - 6 \text{ ft})^3}{3} + (50 \text{ lb})x$$

$$= - \left(\frac{25 \text{ lb}}{9 \text{ ft}} \right) (x - 6 \text{ ft})^3 + (50 \text{ lb})x$$



The plots



Problem 10.47 Determine the shear force V and bending moment M for the beam as functions of x .

Solution: From the free-body diagram of the entire beam we learn that $A_x = A_y = 0$, $M_A = 1800$ ft-lb.

From $x = 0$ to $x = 3$ ft, the distributed load on the beam is

$$w = x \left(\frac{600 \text{ lb/ft}}{3 \text{ ft}} \right) = (200 \text{ lb/ft}^2)x.$$

Using this expression we can integrate Eq. (10.4) to determine V as a function of x .

$$\begin{aligned} \int_0^x dV = V - V_0 &= - \int_0^x w dx = - \int_0^x (200 \text{ lb/ft}^2)x dx \\ &= -(100 \text{ lb/ft}^2)x^2. \end{aligned}$$

The clockwise couple at $x = 0$ causes an increase in the bending moment of 1800 ft-lb. We can integrate Eq. (10.6) to determine M as a function of x .

$$\begin{aligned} \int_0^x dM = M - M_0 &= (1800 \text{ ft-lb}) + \int_0^x V dx = (1800 \text{ ft-lb}) - \int_0^x (100 \text{ lb/ft}^2)x^2 dx \\ &= (1800 \text{ ft-lb}) - (33.3 \text{ lb/ft}^2)x^3. \end{aligned}$$

From $x = 3$ ft to $x = 6$ ft, the distributed load on the beam can be expressed as a linear equations $w = ax + b$. At $x = 3$ ft, $w = -600$ lb/ft, and at $x = 6$ ft, $w = 0$. Using these two conditions to determine a and b , we find that w is given as a function of x by

$$w = (200 \text{ lb/ft}^2)x - 1200 \text{ lb/ft}.$$

To obtain V as a function of x , let us integrate Eq. (10.4) from an arbitrary position x in the range $3 \text{ ft} < x < 6$ ft to $x = 6$ ft.

$$\begin{aligned} \int_x^6 dV &= - \int_x^6 w dx \\ V &= \int_x^6 [(200 \text{ lb/ft}^2)x - 1200 \text{ lb/ft}] dx \\ &= [(100 \text{ lb/ft}^2)x^2 - (1200 \text{ lb/ft})x]_x^6 \\ &= -3600 \text{ lb} + (1200 \text{ lb/ft})x - (100 \text{ lb/ft}^2)x^2. \end{aligned}$$

Then to obtain M as a function of x , we integrate Eq. (10.6) from an arbitrary position x in the range $3 \text{ ft} < x < 6$ ft to $x = 6$ ft.

$$\begin{aligned} \int_x^6 dM &= - \int_x^6 V dx \\ M &= - \int_x^6 [-3600 \text{ lb} + (1200 \text{ lb/ft})x - (100 \text{ lb/ft}^2)x^2] dx \\ &= [(3600 \text{ lb})x - (600 \text{ lb/ft})x^2 - (33.3 \text{ lb/ft}^2)x^3]_x^6 \\ &= (7200 \text{ lb-ft}) - (3600 \text{ lb})x + (600 \text{ lb/ft})x^2 - (33.3 \text{ lb/ft}^2)x^3. \end{aligned}$$

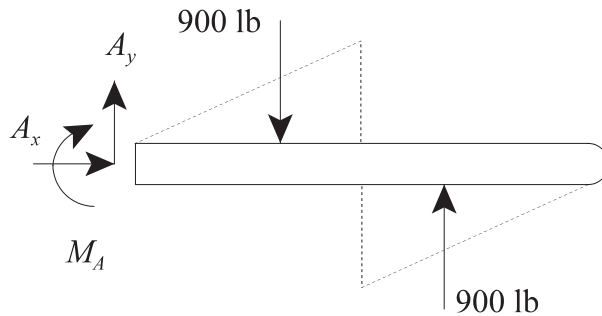
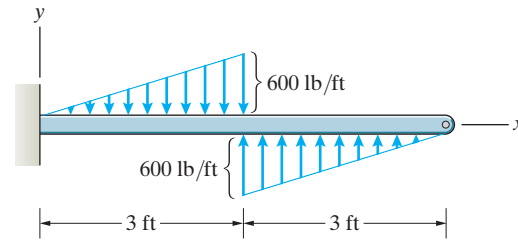
$0 < x < 3 \text{ ft}$

$$V = -(100 \text{ lb/ft}^2)x^2, M = (1800 \text{ ft-lb}) - (33.3 \text{ lb/ft}^2)x^3.$$

$3 \text{ ft} < x < 6 \text{ ft}$

$$V = -(3600 \text{ lb}) + (1200 \text{ lb/ft})x - (100 \text{ lb/ft}^2)x^2,$$

$$M = (7200 \text{ lb-ft}) - (3600 \text{ lb})x + (600 \text{ lb/ft})x^2 - (33.3 \text{ lb/ft}^2)x^3.$$



Problem 10.48* Draw the shear force and bending moment diagrams.

Solution: From 10.40 we have $A = 23.3 \text{ kN}$, $B = 18.67 \text{ kN}$

In the first region $0 < x < 6 \text{ m}$

$$w_1 = 0$$

$$V_1 = -6 \text{ kN}$$

$$M_1 = -(6 \text{ kN})x - 20 \text{ kN-m}$$

In the second region $6 \text{ m} < x < 12 \text{ m}$

$$w_2 = 4 \text{ kN/m}$$

$$V_2 = -(4 \text{ kN/m})(x - 6 \text{ m}) + 17.2 \text{ kN}$$

$$M_2 = -(2 \text{ kN/m})(x - 6 \text{ m})^2 + (17.2 \text{ kN})(x - 6 \text{ m}) - 56 \text{ kN-m}$$

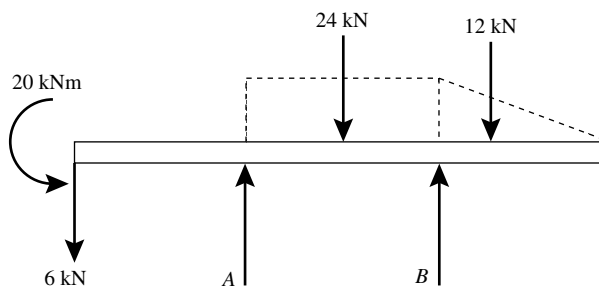
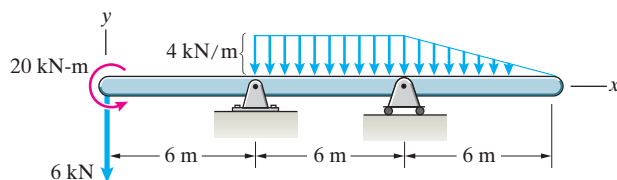
In the last region $12 \text{ m} < x < 18 \text{ m}$

$$w_3 = 4 \text{ kN/m} - \frac{4 \text{ kN/m}}{6 \text{ m}}(x - 12 \text{ m})$$

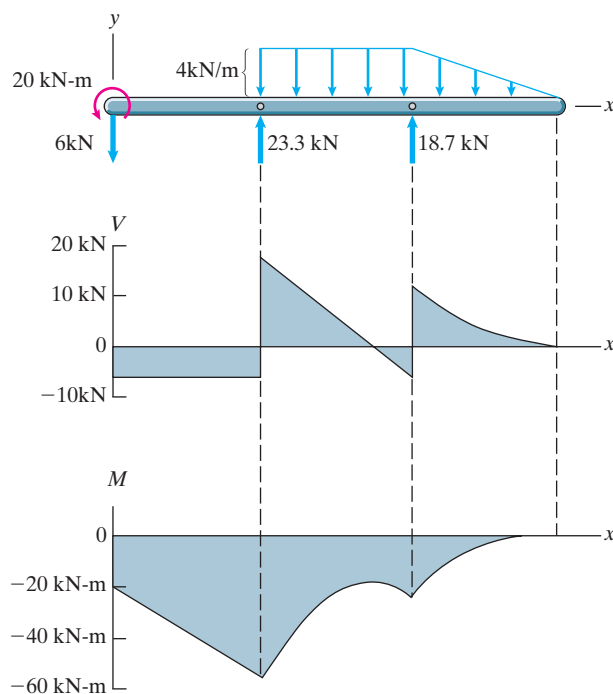
$$= \left(4 \frac{\text{kN}}{\text{m}}\right) - \left(\frac{2 \text{ kN}}{3 \text{ m}^2}\right)(x - 12 \text{ m})$$

$$V_3 = -\left(4 \frac{\text{kN}}{\text{m}}\right)(x - 12 \text{ m}) + \left(\frac{1 \text{ kN}}{3 \text{ m}^2}\right)(x - 12 \text{ m})^2 + 12 \text{ kN}$$

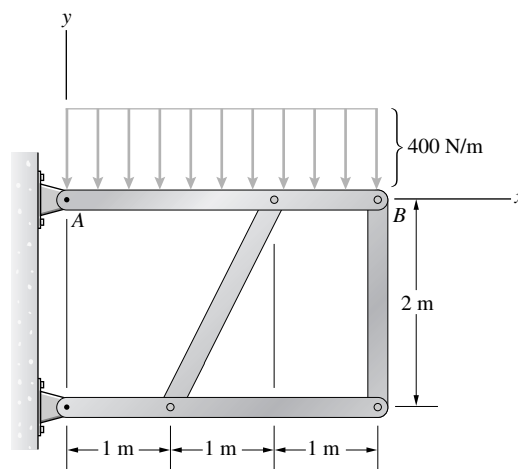
$$M_3 = -\left(2 \frac{\text{kN}}{\text{m}}\right)(x - 12 \text{ m})^2 + \left(\frac{1 \text{ kN}}{9 \text{ m}^2}\right)(x - 12 \text{ m})^3 + (12 \text{ kN})(x - 12 \text{ m}) - 24 \text{ kN-m}$$



The plots



Problem 10.49 Draw the shear force and bending moment diagrams for the beam AB .



Solution: First we must find the reactions

$$\sum M_B : -A_x (2 \text{ m}) - (1200 \text{ N})(1.5 \text{ m})$$

Using free-body diagram, ACD

$$\sum F_x : A_x - \frac{1}{\sqrt{5}} T_C = 0$$

$$\sum M_A : -(1200 \text{ N})(1.5 \text{ m}) - \left(\frac{2}{\sqrt{5}} T_C \right) (2 \text{ m}) - T_D (3 \text{ m}) = 0$$

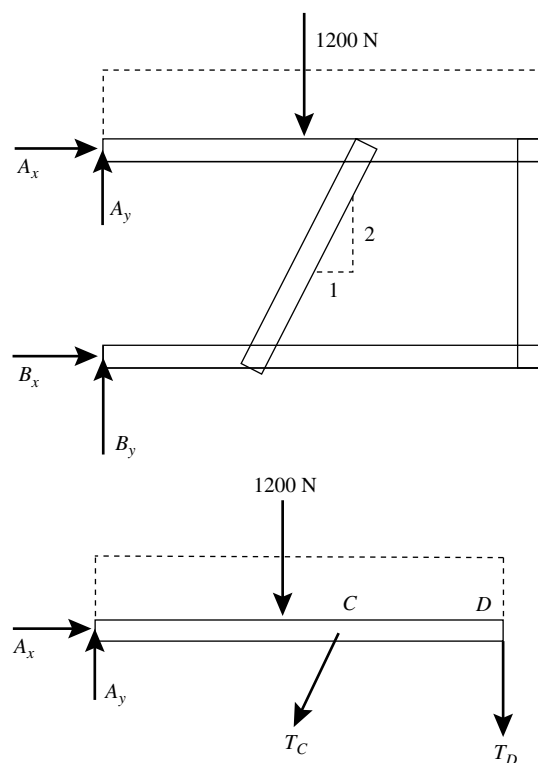
$$\sum F_y : A_y - 1200 \text{ N} - \frac{2}{\sqrt{5}} T_C - T_D = 0$$

We find

$$A_x = -900 \text{ N}, A_y = 0$$

$$T_C = -2012 \text{ N}, T_D = 600 \text{ N}$$

Now we are ready to construct the diagrams.



In the first region $0 < x < 2 \text{ m}$

$$w_1 = 400 \text{ N/m}$$

$$V_1 = -(400 \text{ N/m})x$$

$$M_1 = -(200 \text{ N/m})x^2$$

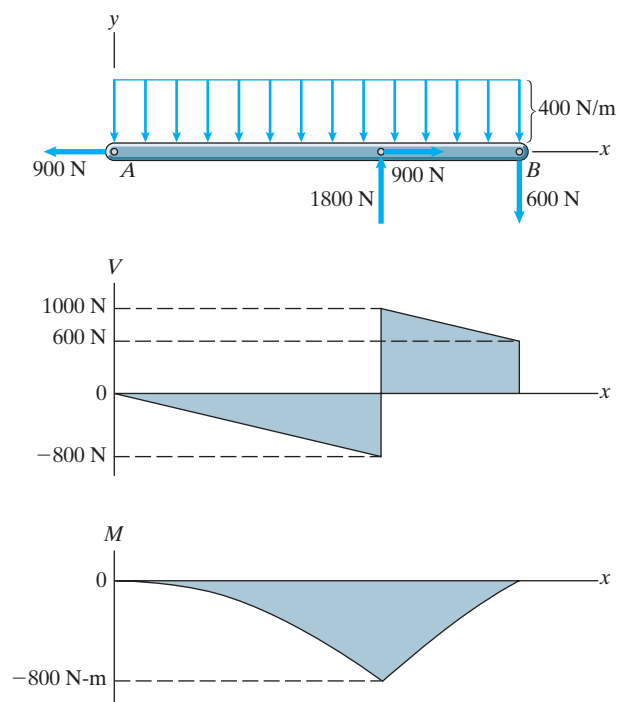
In the second region $2 \text{ m} < x < 3 \text{ m}$

$$w_2 = 400 \text{ N/m}$$

$$V_2 = -(400 \text{ N/m})(x - 2 \text{ m}) + 1800 \text{ N}$$

$$M_2 = -(200 \text{ N/m})(x - 2 \text{ m})^2 + (1800 \text{ N})(x - 2 \text{ m}) - 1600 \text{ N}\cdot\text{m}$$

The plots



Problem 10.50 The cable supports a distributed load $w = 12,000$ lb/ft. Using the approach described in Active Example 10.6, determine the maximum tension in the cable.

Solution: Equation (10.10) must be satisfied for both attachment points:

$$y_L = 40 \text{ ft} = \frac{1}{2}ax_L^2, \quad y_R = 90 \text{ ft} = \frac{1}{2}ax_R^2.$$

Dividing the second equation by the first yields $\frac{x_R^2}{x_L^2} = 2.25$.

The horizontal span of the bridge is $x_R - x_L = 100$ ft.

Solving these two equations yields $x_L = -40$ ft and $x_R = 60$ ft.

Substituting the coordinates of the right attachment point into Eq. (10.10),

$$y_R = \frac{1}{2}ax_R^2 \Rightarrow 90 \text{ ft} = \frac{1}{2}a(60 \text{ ft})^2 \Rightarrow a = 0.05 \text{ ft}^{-1}.$$

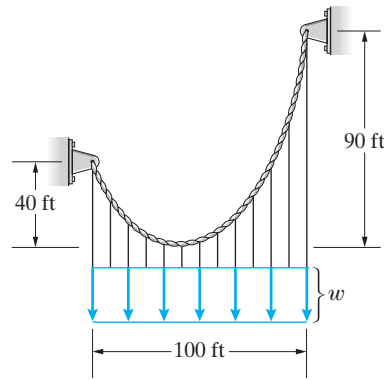
Therefore the tension at the lowest point is

$$T_0 = \frac{w}{a} = \frac{12,000 \text{ lb/ft}}{0.05 \text{ ft}^{-1}} = 240,000 \text{ lb}.$$

The maximum tension in the cable occurs at its right end. From Eq. (10.11),

$$T = T_0 \sqrt{1 + a^2 x^2} = (240 \text{ kip}) \sqrt{1 + (0.05 \text{ ft}^{-1})^2 (60 \text{ ft})^2} = 759 \text{ kip}.$$

759 kip.



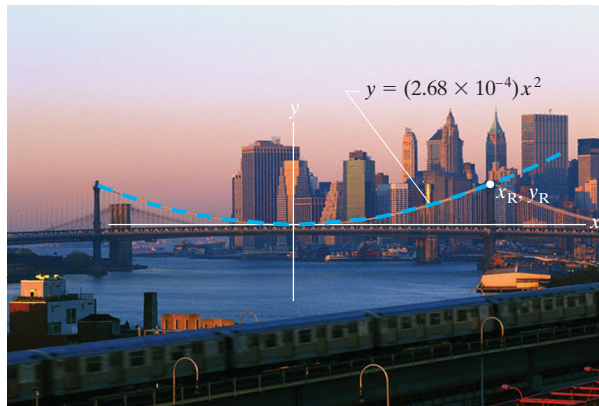
Problem 10.51 In Example 10.7, suppose that the tension at the lowest point of one of the main supporting cables of the bridge is two million pounds. What is the maximum tension in the cable?

Solution:

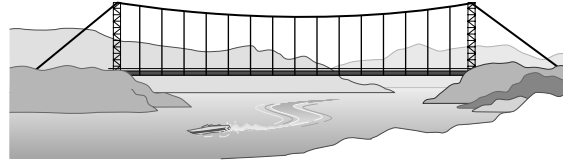
The parameter $a = 5.37 \times 10^{-4} \text{ ft}^{-1}$. The horizontal coordinate of one of the supporting towers relative to the lowest point of the cable is $x_R = 735$ ft. From Eq. (10.11) the maximum tension in the cable is

$$\begin{aligned} T &= T_0 \sqrt{1 + a^2 x^2} \\ &= (2000 \text{ kip}) \sqrt{1 + (5.37 \times 10^{-4} \text{ ft}^{-1})^2 (735 \text{ ft})^2} = 2150 \text{ kip}. \end{aligned}$$

2.15 million pounds.



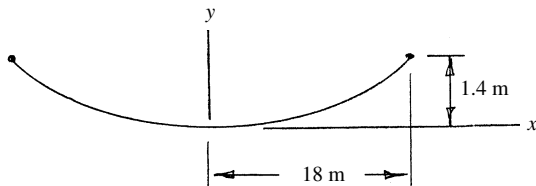
Problem 10.52 A cable is used to suspend a pipeline above a river. The towers supporting the cable are 36 m apart. The lowest point of the cable is 1.4 m below the tops of the towers. The mass of the suspended pipe is 2700 kg.



- (a) What is the maximum tension in the cable?
 (b) What is the suspending cable's length?

Solution: The distributed load is

$$w = \frac{(2700 \text{ kg})(9.81 \text{ m/s}^2)}{36 \text{ m}} = 736 \text{ N/m}.$$



- (a) Setting $x = 18 \text{ m}$, $y = 1.4 \text{ m}$ in Eq. (10.10),

$$1.4 = \frac{1}{2}a(18)^2,$$

we obtain

$$a = \frac{w}{T_0} = 0.00864 \text{ m}^{-1}.$$

Therefore the tension at $x = 0$ is

$$T_0 = \frac{w}{a} = \frac{736}{0.00864} = 85,100 \text{ N}.$$

From Eq. (10.11), the maximum tension is

$$T = T_0\sqrt{1 + a^2(18)^2} = 86,200 \text{ N}.$$

- (b) Setting $x = 18 \text{ m}$ in Eq. (10.12), the length of the cable is

$$\begin{aligned} 2s &= 18\sqrt{1 + a^2(18)^2} + \frac{1}{a} \ln(18a + \sqrt{1 + a^2(18)^2}) \\ &= 36.14 \text{ m}. \end{aligned}$$

Problem 10.53 In Problem 10.52, let the lowest point of the cable be a distance h below the tops of the towers supporting the cable.

- (a) If the cable will safely support a tension of 70 kN, what is the minimum safe value of h ?
 (b) If h has the value determined in part (a), what is the suspending cable's length?

Solution: See the solution of Problem 10.52.

- (a) The distributed load is $w = 736 \text{ N/m}$. Therefore

$$w = 736 = aT_0, \quad (1)$$

And setting $x = 18 \text{ m}$ and $T = 70,000 \text{ N}$ in Eq. (10.11),

$$70,000 = T_0\sqrt{1 + (18)^2a^2}. \quad (2)$$

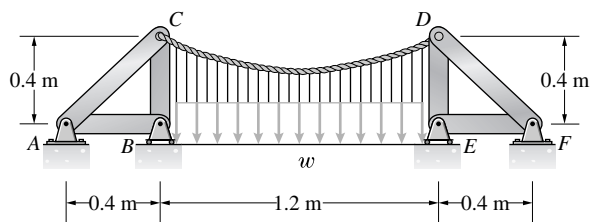
From Eqs. (1) and (2) we obtain $a = 0.0107 \text{ m}^{-1}$, $T_0 = 68,700 \text{ N}$. From Eq. (10.10),

$$\begin{aligned} h &= \frac{1}{2}(0.0107)(18)^2 \\ &= 1.734 \text{ m}. \end{aligned}$$

- (b) From Eq. (10.12), the length of the cable is

$$\begin{aligned} 2s &= 18\sqrt{1 + a^2(18)^2} + \frac{1}{a} \ln(18a + \sqrt{1 + a^2(18)^2}) \\ &= 36.22 \text{ m}. \end{aligned}$$

Problem 10.54 The cable supports a uniformly distributed load $w = 750 \text{ N/m}$. The lowest point of the cable is 0.18 m below the attachment points C and D . Determine the axial loads in the truss members AC and BC .



Solution:

$$y = \frac{1}{2}ax^2 :$$

$$0.18 = \frac{1}{2}a(0.6)^2.$$

From this equation we obtain $a = 1 \text{ m}^{-1}$.

Therefore

$$T_0 = \frac{w}{a} = 750 \text{ N}$$

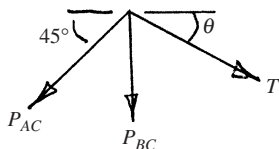
$$\text{and } T = T_0 \sqrt{1 + a^2(0.6)^2} = 875 \text{ N.}$$

From the equation

$$\tan \theta = ax = (1)(0.6),$$

we obtain $\theta = 30.96^\circ$.

The free-body diagram of joint C is shown.



From the equations

$$\sum F_x = T \cos \theta - P_{AC} \cos 45^\circ = 0,$$

$$\sum F_y = -T \sin \theta - P_{BC}$$

$$- P_{AC} \sin 45^\circ = 0,$$

we obtain

$$P_{AC} = 1061 \text{ N,}$$

$$P_{BC} = -1200 \text{ N.}$$

Problem 10.55 The cable supports a railway bridge between two tunnels. The distributed load is $w = 1 \text{ MN/m}$, and $h = 40 \text{ m}$.

- (a) What is the maximum tension in the cable?
 (b) What is the length of the cable?

Solution: The parameter

$$a = 2 \frac{w}{x^2} = 2 \frac{40}{36^2} = 0.06173.$$

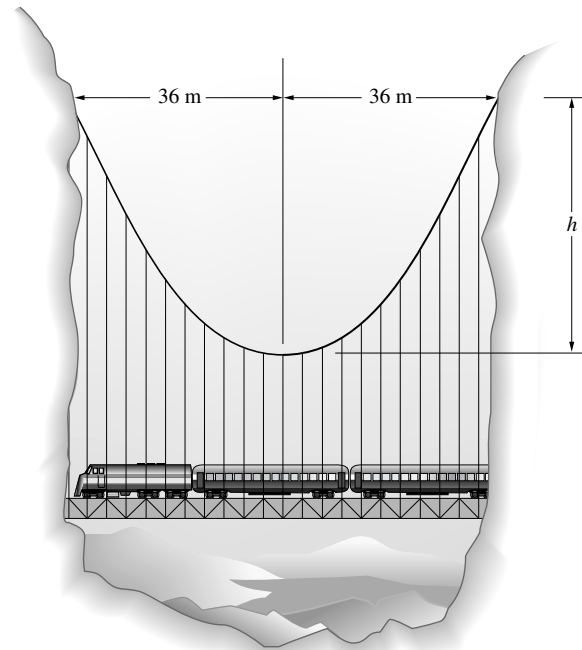
The tension at the lowest point:

$$T_0 = \frac{w}{a} = \frac{1 \times 10^6}{a} = 16200 \text{ kN}.$$

The maximum tension: $T_{\text{MAX}} = T_0 \sqrt{1 + a^2 x^2}$, which, for $x = 36 \text{ m}$, $T_{\text{MAX}} = 39477 \text{ kN}$. The cable length is

$$s(x) = \left(x \sqrt{1 + a^2 x^2} + \frac{1}{a} \ln \left(ax + \sqrt{1 + a^2 x^2} \right) \right),$$

which, for $x = 36 \text{ m}$, $L = 112.66 \text{ m}$.



Problem 10.56 The cable in Problem 10.55 will safely support a tension of 40 MN. What is the shortest cable that can be used, and what is the corresponding value of h ?

Solution: The tension at the lowest point is

$$T_0 = \frac{w}{a}.$$

The maximum tension is

$$T_{\text{MAX}} = T_0 \sqrt{1 + a^2 x^2}.$$

Square both sides, substitute and reduce algebraically: $T_o^2 = T_{\text{MAX}}^2 - w^2 x^2$. The terms on the right are known: $T_{\text{MAX}}^2 = 40^2 (10^6)$, and $w^2 x^2 = 36^2 (10^6)$. Solve for the parameter a ,

$$a^2 = \frac{10^6}{(40^2 - 36^2)(10^6)} = 3.29 \times 10^{-3},$$

from which $a = 0.0574$. The height is

$$h = \left(\frac{1}{2} \right) ax^2 = \left(\frac{1}{2} \right) (0.0574)(36^2) = 37.165 \text{ m}.$$

The length is

$$s(x) = \left(x \sqrt{1 + a^2 x^2} + \frac{1}{a} \ln \left(ax + \sqrt{1 + a^2 x^2} \right) \right),$$

which, for $x = 36 \text{ m}$, $L = 108.26 \text{ m}$

Problem 10.57 An oceanographic research ship tows an instrument package from a cable. Hydrodynamic drag subjects the cable to a uniformly distributed force $w = 2 \text{ lb/ft}$. The tensions in the cable at 1 and 2 are 800 lb and 1300 lb, respectively. Determine the distance h .

Solution: If one assumes that the cable is tangent to the vertical at the point 1, so that the 800 lb is the tension at the lowest point, the data is inconsistent; therefore the point 1 must be at a distance x_1 from the lowest point. There are three unknowns in the problem: the distance x_1 , the tension at the lowest point T_0 , and the parameter a . The three equations that define these unknowns are:

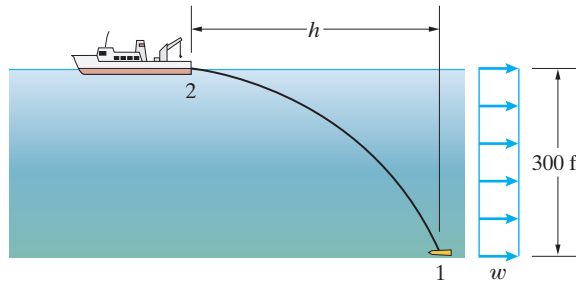
- (1) $800 = T_0 \sqrt{1 + a^2 x_1^2}$,
- (2) $T_0 = \frac{w}{a} = \frac{2}{a}$,
- (3) $1300 = T_0 \sqrt{1 + a^2 (x_1 + 300)^2}$.

These are reduced to two equations in two unknowns:

- (1) $800 = \left(\frac{2}{a}\right) \sqrt{1 + a^2 x_1^2}$,
- (2) $1300 = \left(\frac{2}{a}\right) \sqrt{1 + a^2 (x_1 + 300)^2}$ and solved by iteration using

TK Solver Plus. The result: $a = 3.596 \times 10^{-3}$, $x_1 = 287.5 \text{ ft}$. Using these values, the distance is

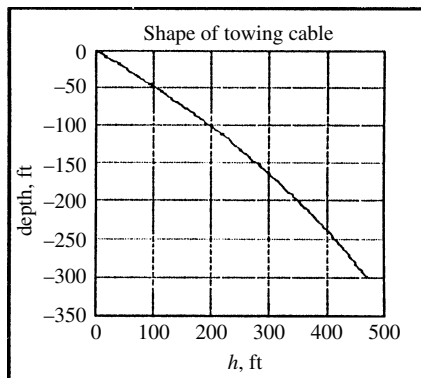
$$h = \left(\frac{1}{2}\right) a (x_1 + 300)^2 - \left(\frac{1}{2}\right) a x_1^2 = 471.94 \text{ ft}$$



Problem 10.58 Draw a graph of the shape of the cable in Problem 10.57.

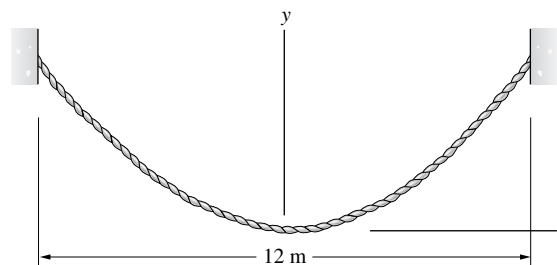
Solution: The following equations are graphed:

- (1) IF $d > x_1$ then $w = h - \left(\frac{1}{2}\right) a d^2 + \left(\frac{1}{2}\right) a x_1^2$,
- (2) $z = 300 + x_1 - d$, where $h = 471.9 \text{ ft}$, $a = 3.59573 \times 10^{-3}$, $x_1 = 287.5 \text{ ft}$. The value w is plotted on the abscissa, and z is plotted on the ordinate. The result is a graph of the depth of the cable against the horizontal extension.



Problem 10.59 The mass of the rope per unit length is 0.10 kg/m. The tension at its lowest point is 4.6 N. Using the approach described in Active Example 10.8, determine

- (a) the maximum tension in the rope
- (b) the rope's length.



Solution: The weight per unit length is

$$w = (0.1 \text{ kg/m})(9.81 \text{ m/s}^2) = 0.981 \text{ N/m}.$$

$$(a) \quad a = \frac{w}{T_0} = \frac{0.981}{4.6} = 0.213 \text{ m}^{-1}.$$

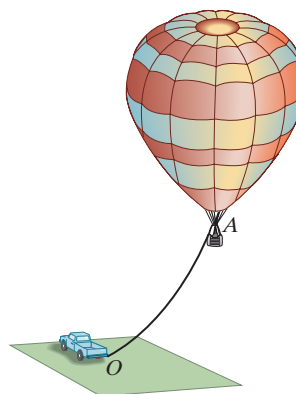
From Eq. (10.21), the maximum tension is

$$\begin{aligned} T &= T_0 \cosh(ax) \\ &= 4.6 \cosh[(0.213)(6)] \\ &= 8.91 \text{ N}. \end{aligned}$$

(b) From Eq. (10.22), the length is

$$\begin{aligned} 2s &= \frac{2 \sinh[(0.213)(6)]}{0.213} \\ &= 15.55 \text{ m}. \end{aligned}$$

Problem 10.60 The stationary balloon's tether is horizontal at point O where it is attached to the truck. The mass per unit length of the tether is 0.45 kg/m. The tether exerts a 50-N horizontal force on the truck. The horizontal distance from point O to point A where the tether is attached to the balloon is 20 m. What is the height of point A relative to point O ?



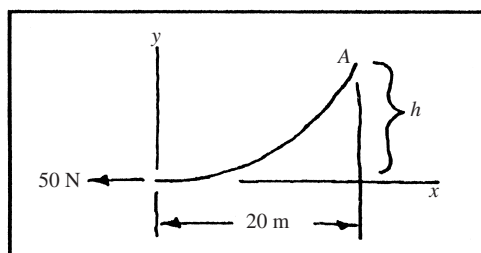
Solution:

$$a = \frac{w}{T_0} = \frac{(9.81)(0.45)}{50} = 0.0883 \text{ m}^{-1}.$$

From equation (10.20),

$$y = \frac{1}{a} [\cosh(ax) - 1]$$

$$h = \frac{1}{0.0883} \{ \cosh[(0.0883)(20)] - 1 \} = 22.8 \text{ m}.$$



Problem 10.61 In Problem 10.60, determine the magnitudes of the horizontal and vertical components of the force exerted on the balloon at A by the tether.

Solution: From the solution to Problems 10.60, $a = 0.0883 \text{ m}^{-1}$. The value of the tension at $x = 20 \text{ m}$ is

$$T = T_0 \cosh(ax) = 50 \cosh[(0.0883)(20)] = 150 \text{ N.}$$

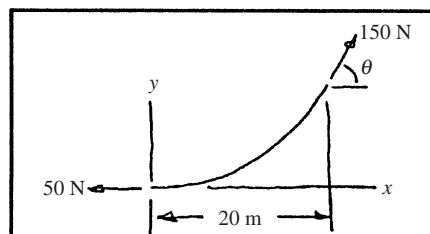
The slope at $x = 20 \text{ m}$ (Equation 10.19) is

$$\sigma = \tan \theta = \sinh(ax) = \sinh[(0.0883)(20)] = 2.84,$$

so $\theta = \arctan 2.84 = 70.6^\circ$. The horizontal and vertical components are

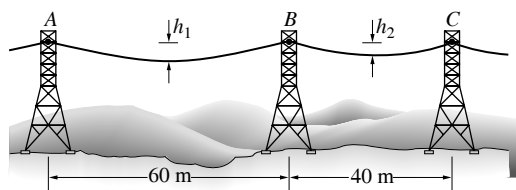
$$T_x = (150) \cos \theta = 50 \text{ N}$$

$$T_y = (150) \sin \theta = 142 \text{ N.}$$



Problem 10.62 The mass per unit length of lines AB and BC is 2 kg/m . The tension at the lowest point of cable AB is 1.8 kN . The two lines exert equal horizontal forces at B.

- Determine the sags h_1 and h_2 .
- Determine the maximum tensions in the two lines.



Solution: The lines meet the condition for a catenary. (a) *The line AB.* The weight density is

$$w = 2(9.81) = 19.62 \text{ N/m.}$$

The parameter

$$a_1 = \frac{w}{T_{AB}} = \frac{19.62}{1800} = 0.0109.$$

The sag is

$$h_1 = \left(\frac{1}{a_1} \right) (\cosh(30a_1) - 1) = 4.949 \text{ m.}$$

The line BC. The horizontal component of the tension at B is $T_{AB} = 1.8 \text{ kN}$. Thus the tension at the lowest point in BC is 1.8 kN , and the parameter a for line BC is equal to a_1 . The sag is

$$h_2 = \left(\frac{1}{a_1} \right) (\cosh(20a_1) - 1) = 2.189 \text{ m.}$$

(b) *The line AB.* The maximum tension is

$$T_{\text{MAX}}^{AB} = T_{AB} \cosh(30a_1) = 1897.1 \text{ N.}$$

The line BC. The maximum tension is

$$T_{\text{MAX}}^{BC} = T_{AB} \cosh(20a_1) = 1842.9 \text{ N.}$$

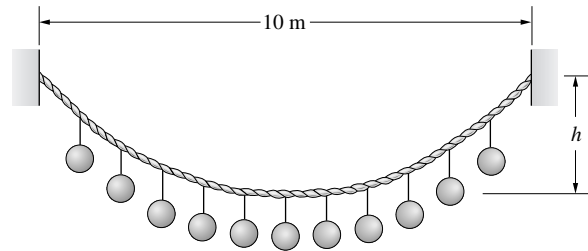
Problem 10.63 The rope is loaded by 2-kg masses suspended at 1-m intervals along its length. The mass of the rope itself is negligible. The tension in the rope at its lowest point is 100 N. Determine h and the maximum tension in the rope.

Strategy: Obtain an approximate answer by modeling the discrete loads on the rope as a load uniformly distributed along its length.

Solution: The equivalent distributed load is

$$w = \frac{(2 \text{ kg})(9.81 \text{ m/s}^2)}{1 \text{ m}} = 19.62 \text{ N/m}.$$

$$\text{Therefore } a = \frac{w}{T_0} = \frac{19.62}{100} = 0.196 \text{ m}^{-1}.$$



From Eq. (10.20),

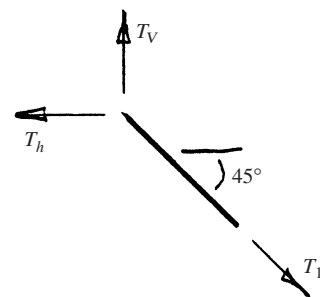
$$h = \frac{1}{a} \{ \cosh[a(5)] - 1 \} = 2.66 \text{ m}.$$

From Eq. (10.21), the maximum tension is

$$T = T_0 \cosh ax = (100) \cosh[a(5)] = 152 \text{ N}.$$

Problem 10.64 In Active Example 10.9, what are the tensions in cable segments 1 and 3?

Solution: See the solution of Example 10.9. Cutting cable segment 1, we obtain the free-body diagram

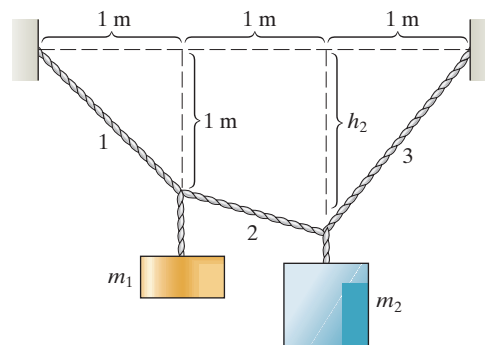
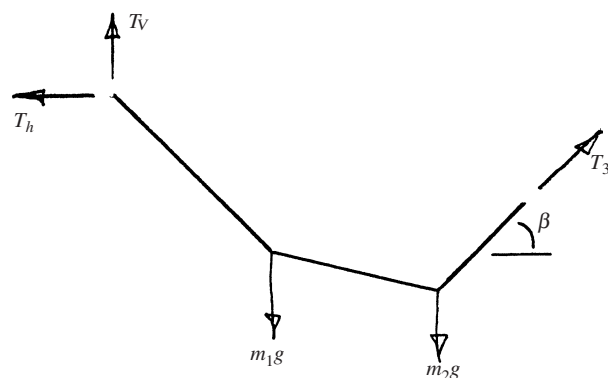


From the equation

$$\sum F_x = -T_h + T_1 \cos 45^\circ = 0,$$

$$\text{we obtain } T_1 = \frac{T_h}{\cos 45^\circ} = \frac{131}{\cos 45^\circ} = 185 \text{ N}.$$

Cutting cable segment 3, we obtain the free-body diagram



The angle β is

$$\beta = \arctan\left(\frac{h_2}{1}\right) = \arctan(1.25) = 51.3^\circ.$$

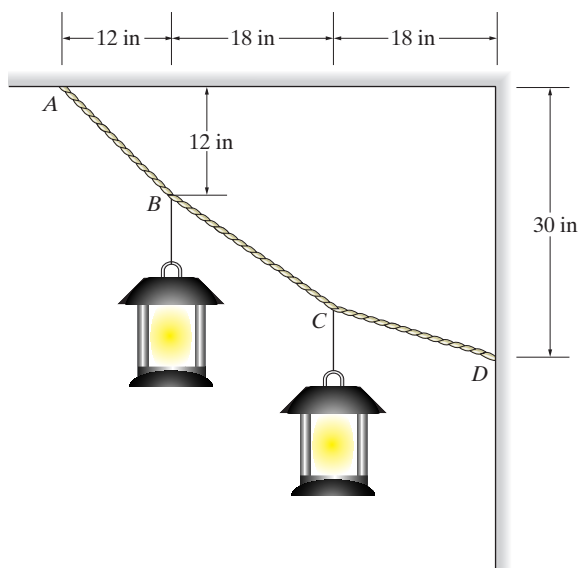
From the equation

$$\sum F_x = -T_h + T_3 \cos \beta = 0,$$

we obtain

$$T_3 = \frac{T_h}{\cos \beta} = \frac{131}{\cos 51.3^\circ} = 209 \text{ N}.$$

- Problem 10.65** Each lamp weighs 12 lb.
 (a) What is the length of the wire $ABCD$ needed to suspend the lamps as shown?
 (b) What is the maximum tension in the wire?



Solution:

$$\sum M_B : -T_V(12 \text{ in}) + T_H(12 \text{ in}) = 0 \Rightarrow T_V = T_H$$

Using the second free-body diagram shown,

$$\sum M_C : T_H h - T_V(30 \text{ in}) + (12 \text{ lb})(18 \text{ in}) = 0$$

Make a cut at the right attachment point and take moments

$$\sum M_D : T_H(30 \text{ in}) - T_V(48 \text{ in}) + (12 \text{ lb})(36 \text{ in}) + (12 \text{ lb})(18 \text{ in}) = 0$$

Solving together we find

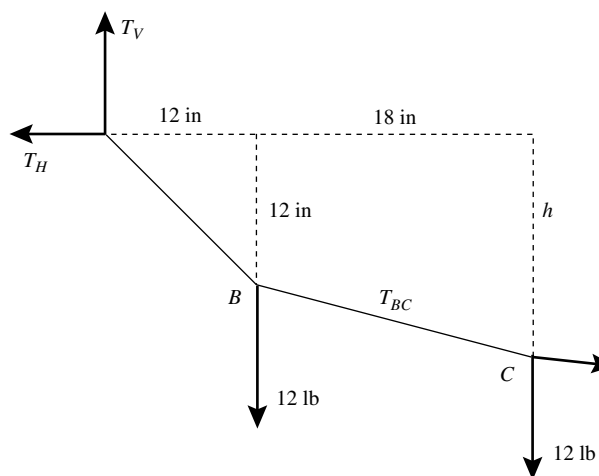
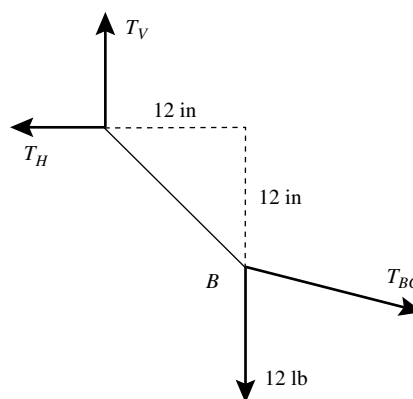
$$T_H = T_V = 36 \text{ lb}, h = 24 \text{ in}$$

- (a) The length of the cable is then

$$L = \sqrt{12^2 + 12^2} + \sqrt{18^2 + (24 - 12)^2} + \sqrt{18^2 + (30 - 24)^2} = 57.6 \text{ in}$$

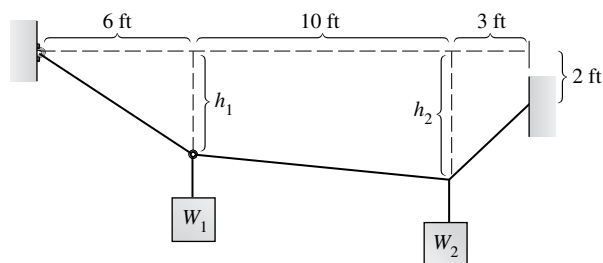
- (b) The maximum tension occurs where the angle is the greatest (AB)

$$T_{\max} = T_{AB} = \sqrt{36^2 + 36^2} = 50.9 \text{ lb}$$



Problem 10.66 Two weights, $W_1 = W_2 = 50$ lb, are suspended from a cable. The vertical distance $h_1 = 4$ ft.

- Determine the vertical distance h_2 .
- What is the maximum tension in the cable?



Solution: The strategy is to make cuts along the string and sum the moments to the left of each cut. The three simultaneous equations are then solved for the unknowns. Make a cut at left attachment and to the right of W_1 . Denote the components of the force exerted by the string by F_H and F_V , with sign indicating direction. The sum of the moments about the right end:

$$\sum M = -6F_V - h_1F_H = 0.$$

Make the cut to the right of W_2 . The sum of the moments about the right end:

$$\sum M = -h_2F_H - 16F_V + 10W_1 = 0.$$

Make the cut at the right attachment point and sum the moments to the left:

$$\sum M = -19F_V - h_2F_H + 13W_1 + 3W_2 = 0.$$

Solve:

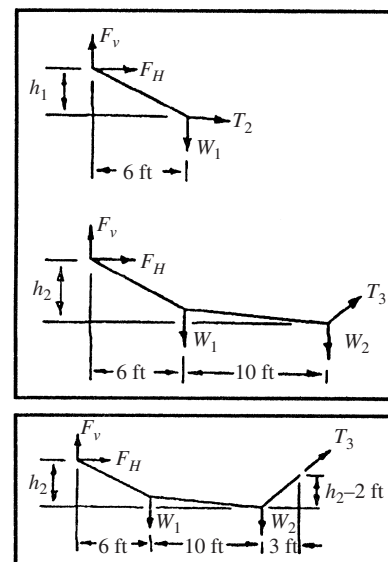
$$h_2 = 4 \text{ ft},$$

$$F_V = 50 \text{ lb},$$

$$\text{and } F_H = -75 \text{ lb}.$$

From the sum of the forces for the complete string the tension in the right support string is equal to the tension in the string. Thus the maximum tension is

$$T = \sqrt{F_V^2 + F_H^2} = 90.14 \text{ lb}.$$



Problem 10.67 In Problem 10.66, $W_1 = 50$ lb, $W_2 = 100$ lb, and the vertical distance $h_1 = 4$ ft.

- Determine the vertical distance h_2 .
- What is the maximum tension in the cable?

Solution: (a) Cut the cable at the left and just to the right of w_1 :

$$\sum M_{(\text{rightend})} = h_1 T_h - 6T_v = 0 \quad (1)$$

Cut the cable at the left and just to the right of w_2 :

$$\sum M_{(\text{rightend})} = h_2 T_h - 16T_v + 10w_1 = 0 \quad (2).$$

Cut the cable at the left and right:

$$\sum M_{(\text{rightend})} = 2T_h - 19T_v + 13w_1 + 3w_2 = 0. \quad (3)$$

Knowing

$$W_1 = 50 \text{ lb.}$$

$$W_2 = 100 \text{ lb}$$

and $h_1 = 4$ ft,

equations (1), (2) and (3) can be solved for T_h , T_v and h_2 , obtaining

$$T_h = 89.1 \text{ lb,}$$

$$T_v = 59.4 \text{ lb,}$$

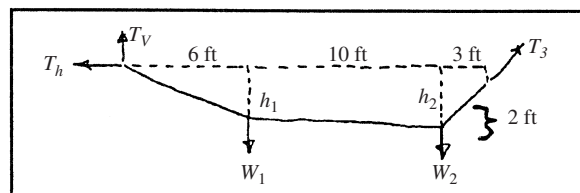
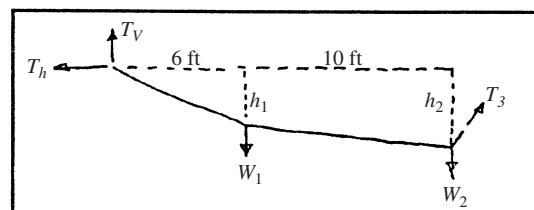
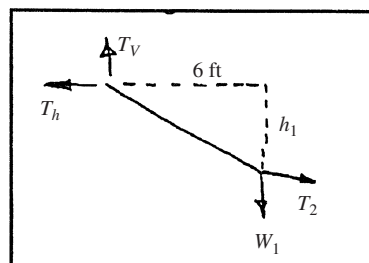
$$h_2 = 5.05 \text{ ft.}$$

(b) The maximum tension occurs in the segment with the largest slope relative to the horizontal. In this problem T_3 is the largest tension. The angle between T_3 and the horizontal: is

$$\theta = \arctan \left(\frac{h_2 - 2}{3} \right) = 45.5^\circ.$$

Summing horizontal forces on the third free body diagram, we obtain $-T_h + T_3 \cos \theta = 0$, so

$$T_3 = \frac{T_h}{\cos \theta} = \frac{89.1}{\cos 45.5^\circ} = 127 \text{ lb.}$$



Problem 10.68 Three identical masses $m = 10 \text{ kg}$ are suspended from the cable. Determine the vertical distances h_1 and h_3 and draw a sketch of the configuration of the cable.

Solution: We make 3 cuts and then draw one diagram of the whole system

$$\sum M_{A_1} : T_H h_1 - T_V (2 \text{ m}) = 0$$

$$\sum M_{A_2} : T_H (2 \text{ m}) - T_V (3 \text{ m}) + (98.1 \text{ N})(1 \text{ m}) = 0$$

$$\sum M_{A_3} : T_H h_3 - T_V (6 \text{ m})$$

$$+ (98.1 \text{ N})(4 \text{ m}) + (98.1 \text{ N})(3 \text{ m}) = 0$$

$$\sum M_{A_4} : -T_V (7 \text{ m}) + (98.1 \text{ N})(5 \text{ m})$$

$$+ (98.1 \text{ N})(4 \text{ m}) + (98.1 \text{ N})(1 \text{ m}) = 0$$

Solving we find

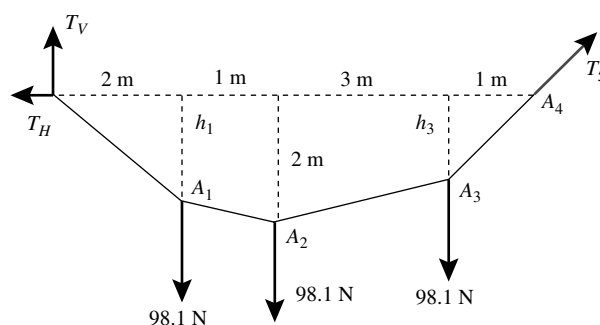
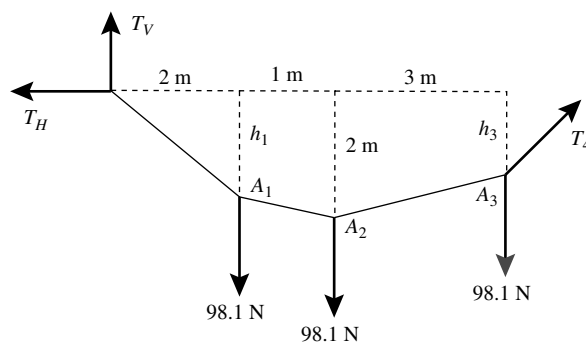
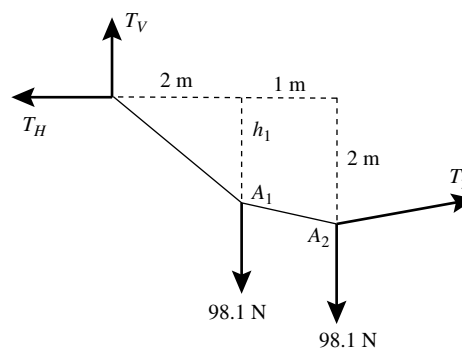
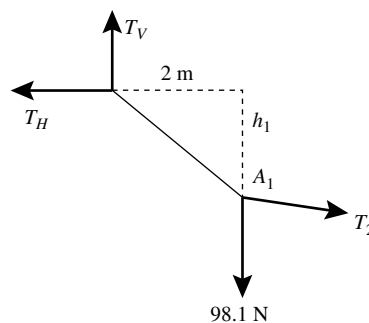
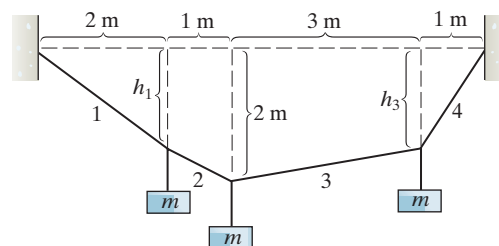
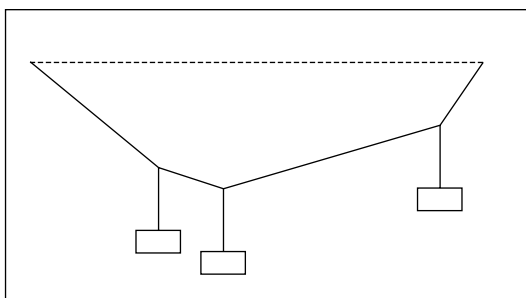
$$h_1 = 1.739 \text{ m}$$

$$h_3 = 0.957 \text{ m}$$

$$T_H = 161.2 \text{ N}$$

$$T_V = 140.1 \text{ N}$$

Sketch the configuration



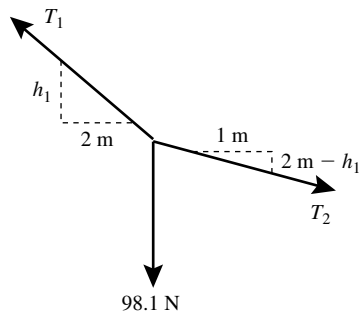
Problem 10.69 In Problem 10.68, what are the tensions in cable segments 1 and 2?

Solution: Use the solution to 10.68

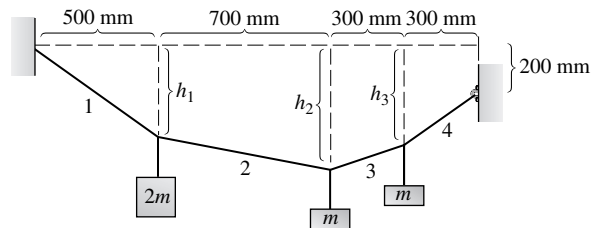
$$\sum F_x : -\frac{2 \text{ m}}{\sqrt{(2 \text{ m})^2 + h_1^2}} T_1 + \frac{1 \text{ m}}{\sqrt{(1 \text{ m})^2 + (2 \text{ m} - h_1)^2}} T_2 = 0$$

$$\sum F_y : \frac{h_1}{\sqrt{(2 \text{ m})^2 + h_1^2}} T_1 - \frac{2 \text{ m} - h_1}{\sqrt{(1 \text{ m})^2 + (2 \text{ m} - h_1)^2}} T_2 - 98.1 \text{ N} = 0$$

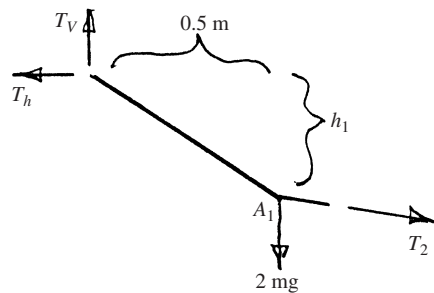
Solving $T_1 = 214 \text{ N}$, $T_2 = 167 \text{ N}$



Problem 10.70 Three masses are suspended from the cable, where $m = 30 \text{ kg}$, and the vertical distance $h_1 = 400 \text{ mm}$. Determine the vertical distances h_2 and h_3 .



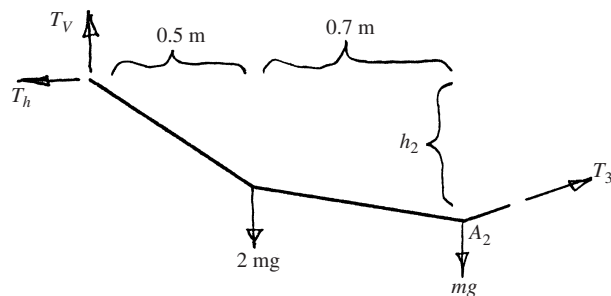
Solution: Cutting to the right of the left mass,



we obtain

$$\sum M_{(pt A_1)} = h_1 T_h - (0.5) T_n = 0. \quad (1)$$

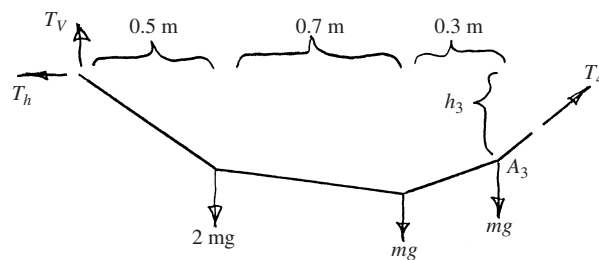
Cutting to the right of the middle mass,



we obtain

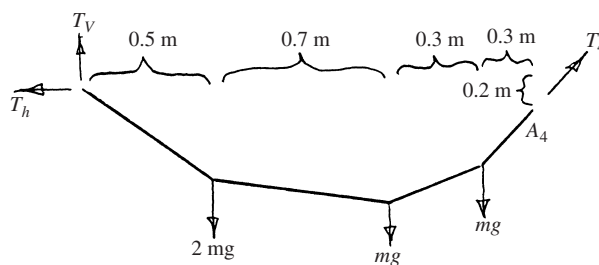
$$\sum M_{(pt A_2)} = h_2 T_h - (1.2) T_n + (0.7) 2 \text{ mg} = 0. \quad (2)$$

Cutting to the right of the right mass, we obtain



$$\sum M_{(pt A_3)} = h_3 T_h - (1.5) T_n + (1) 2 \text{ mg} + (0.3) \text{ mg} = 0. \quad (3)$$

Finally, cutting at the right attachment point,



we obtain

$$\sum M_{(pt A_4)} = (0.2) T_h - (1.8) T_n + (1.3) 2 \text{ mg} + (0.6) \text{ mg} + (0.3) \text{ mg} = 0. \quad (4)$$

Solving Equations (1)–(4), we obtain $T_h = 831 \text{ N}$, $T_n = 665 \text{ N}$, $h_2 = 464 \text{ mm}$, $h_3 = 385 \text{ mm}$.

Problem 10.71 In Problem 10.70, what is the maximum tension in the cable, and where does it occur?

Solution: The tension is greatest in the segment with the greatest slope, which is either segment 1 or segment 4.

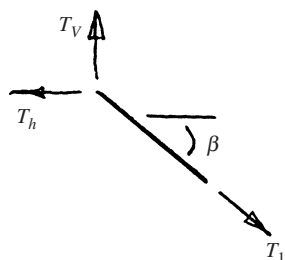
Slope of segment 1 (see the solution of Problem 10.71):

$$\frac{h_1}{500} = \frac{400}{500} = 0.8.$$

Slope of segment 4:

$$\frac{h_3 - 200}{300} = \frac{385 - 200}{300} = 0.62.$$

Cutting segment 1, we obtain



The angle β is

$$\beta = \arctan\left(\frac{h_1}{500}\right) = \arctan\left(\frac{400}{500}\right) = 38.7^\circ.$$

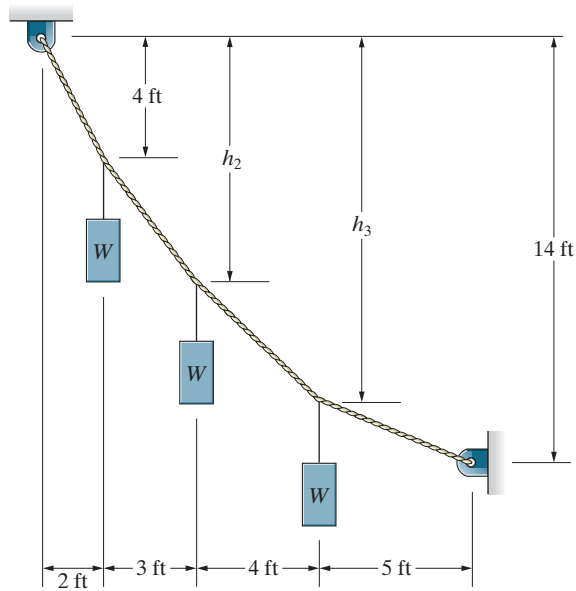
From the equation

$$\sum F_x = -T_h + T_1 \cos \beta = 0,$$

we obtain

$$T_1 = \frac{T_h}{\cos \beta} = \frac{831}{\cos 38.7^\circ} = 1060 \text{ N}.$$

Problem 10.72 Each suspended object has the same weight W . Determine the vertical distances h_2 and h_3 .



Solution:

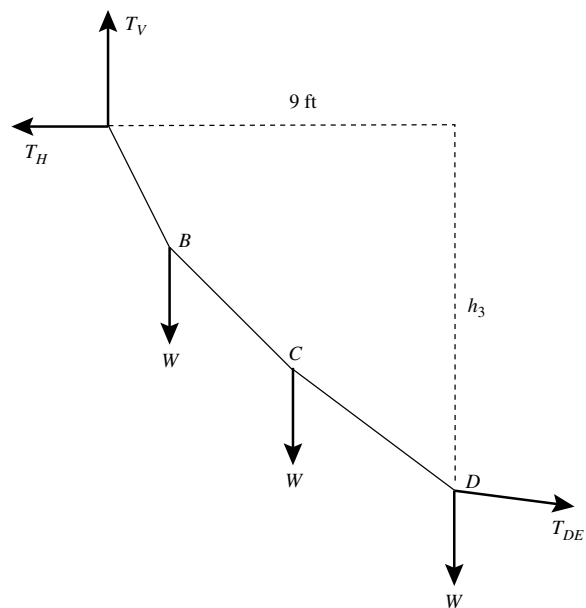
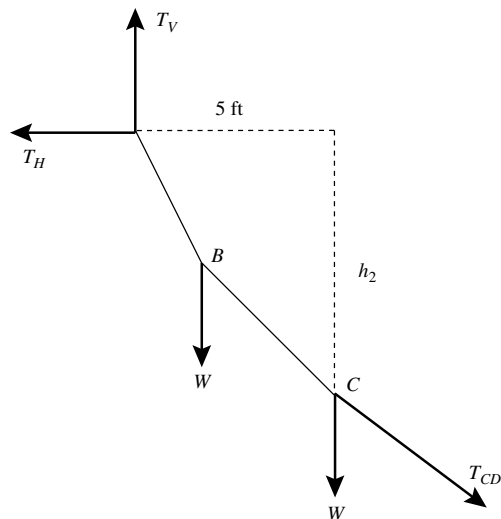
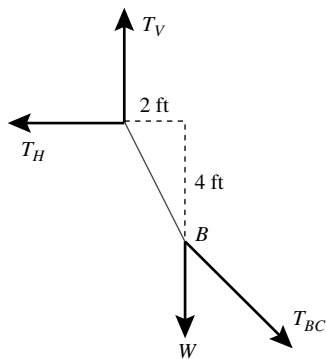
$$\sum M_B : -T_V(2 \text{ ft}) + T_H(4 \text{ ft}) = 0$$

$$\sum M_C : -T_V(5 \text{ ft}) + T_H h_2 + W(3 \text{ ft}) = 0$$

$$\sum M_D : -T_V(9 \text{ ft}) + T_H h_3 + W(7 \text{ ft}) + W(4 \text{ ft}) = 0$$

$$\sum M_E : -T_V(14 \text{ ft}) + T_H(14 \text{ ft}) + W(12 \text{ ft}) + W(9 \text{ ft}) + W(5 \text{ ft}) = 0$$

Solving we find $h_2 = 8.38 \text{ ft}, h_3 = 12.08 \text{ ft}$



Problem 10.73 An engineer planning a water system for a new community estimates that at maximum expected usage, the pressure drop between the central system and the farthest planned fire hydrant will be 25 psi. Fire fighting personnel indicate that a gage pressure of 40 psi at the fire hydrant is required. The weight density of the water is $\gamma = 62.4 \text{ lb/ft}^3$. How tall would a water tower at the central system have to be to provide the needed pressure?

Solution: The total pressure will be $65 \left(\frac{\text{lb}}{\text{in}^2} \right)$. A tower of height h will produce a pressure of $P = \gamma h$, where the units are to be consistent. The tower height is $h = \left(\frac{P}{\gamma} \right)$.

Let the tower height be in feet:

$$h \text{ ft} = \left(\frac{1}{62.4} \right) \left(\frac{\text{ft}^3}{\text{lb}} \right) (65) \left(\frac{\text{lb}}{\text{in}^2} \right) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right)^2 = 150 \text{ ft}$$

Problem 10.74 A cube of material is suspended below the surface of a liquid of weight density γ . By calculating the forces exerted on the faces of the cube by pressure, show that their sum is an upward force of magnitude γb^3 .

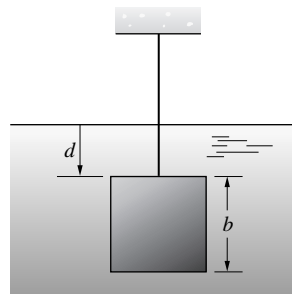
Solution: Neglect the pressures on the supporting wire. The force on the top surface is $F_{\text{TOP}} = \gamma d b^2$. The force on the sides is

$$F_S = \gamma \left(d + \frac{b}{2} \right) b^2,$$

where $\left(d + \frac{b}{2} \right)$

is the distance of the centroid of the area from the surface. The force on the bottom surface is $F_{\text{BOTTOM}} = \gamma(d + b)b^2$. The forces on the sides cancel by symmetry. The difference between the downward force on the top surface and the upward force on the bottom surface is the resultant:

$$R = \gamma(d + b)b^2 - \gamma d b^2 = \gamma b^3.$$



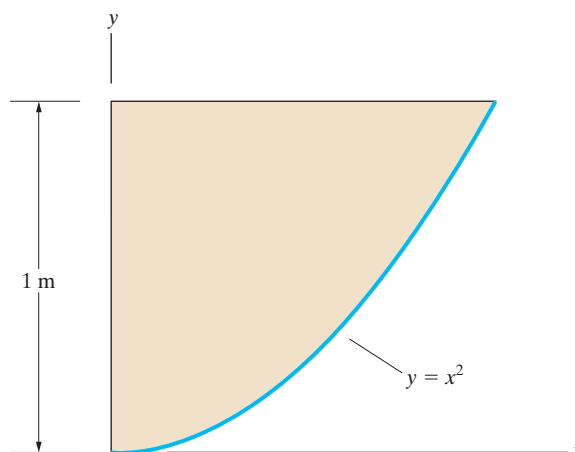
Problem 10.75 The area shown is subjected to a *uniform* pressure $p_{\text{atm}} = 1 \times 10^5 \text{ Pa}$.

- What is the total force exerted on the area by the pressure?
- What is the moment about the y axis due to the pressure?

Solution:

$$(a) \quad F = \int_0^1 \int_{x^2}^1 10^5 \frac{\text{N}}{\text{m}^2} dy dx = 66,700 \text{ N} = 66.7 \text{ kN}$$

$$(b) \quad M_y = \int_0^1 \int_{x^2}^1 x \left(10^5 \frac{\text{N}}{\text{m}^2} \right) dy dx = 25,000 \text{ Nm} = 25 \text{ kN-m}$$



Problem 10.76 The area shown is subjected to a *uniform* pressure. Determine the coordinates of the center of pressure.

Solution:

$$R = \int_0^1 \int_{x^2}^1 P dy dx = \frac{2P}{3}$$

$$M_y = \int_0^1 \int_{x^2}^1 xP dy dx = \frac{P}{4} \Rightarrow \begin{cases} \bar{x} = \frac{M_y}{R} = 0.375 \text{ m} \\ \bar{y} = \frac{M_x}{R} = 0.6 \text{ m} \end{cases}$$

$$M_x = \int_0^1 \int_{x^2}^1 yP dy dx = \frac{2P}{5}$$

Problem 10.77 The area shown is subjected to a *uniform* pressure $p_{\text{atm}} = 14.7$ psi.

- (a) What is the total force exerted on the area by the pressure?
 (b) What is the moment about the y axis due to the pressure on the area?

Solution: (a) The total force is

$$F = \int_A p_{\text{atm}} dA = p_{\text{atm}} \int_A dA = p_{\text{atm}} A$$

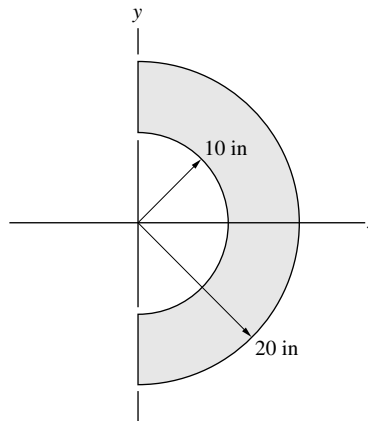
$$= 14.7 \frac{\text{lb}}{\text{in}^2} \left[\frac{1}{2} \pi (20)^2 - \frac{1}{2} \pi (10)^2 \right] \text{in}^2 = 6930 \text{ lb.}$$

(b) We can represent the pressure by an equivalent force F acting at the center of pressure. Since the pressure is uniform, the center of pressure is at the centroid of the plane area. From Appendix b, the x coordinate of the centroid is

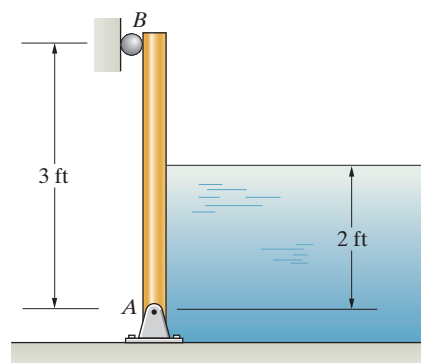
$$x = \frac{\left[\frac{4(20)}{3\pi} \right] \left[\frac{1}{2} \pi (20)^2 \right] - \left[\frac{4(10)}{3\pi} \right] \left[\frac{1}{2} \pi (10)^2 \right]}{\frac{1}{2} \pi (20)^2 - \frac{1}{2} \pi (10)^2} = 9.90 \text{ in.}$$

The moment about the y axis is

$$xF = (9.90)(6930) = 68,600 \text{ in-lb.}$$



Problem 10.78 In Active Example 10.10, suppose that the water depth relative to point A is increased from 2 ft to 3 ft. Determine the reactions on the gate at the supports A and B.



Solution: The gage pressure $p_g = \gamma x$ increases linearly from $p_g = 0$ at the surface to $p_g = \gamma(3 \text{ ft})$ at the bottom of the gate. The centroid of the distribution is located at $\frac{2}{3}(3 \text{ ft}) = 2 \text{ ft}$ below point B.

The force exerted on the gate by the gage pressure is the “volume” of the pressure distribution

$$F = \frac{1}{2}(3 \text{ ft}) \left[(62.4 \text{ lb/ft}^3)(3 \text{ ft}) \right] (3 \text{ ft})$$

$$= 842 \text{ lb.}$$

The equilibrium equations for the gate are

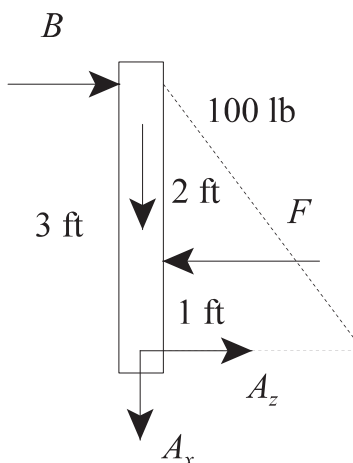
$$\Sigma F_x : A_x + 100 \text{ lb} = 0,$$

$$\Sigma F_z : A_z + B - 842 \text{ lb} = 0,$$

$$\Sigma M_{y \text{ axis}} : A_z(3 \text{ ft}) - (842 \text{ lb})(2 \text{ ft}) = 0.$$

Solving yields

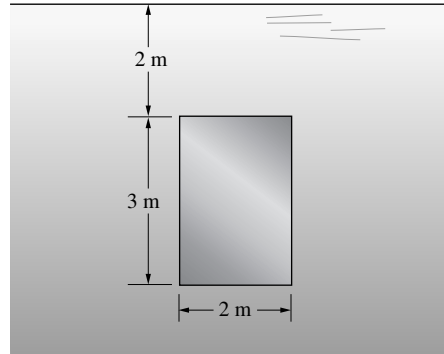
$$A_x = -100 \text{ lb}, A_z = 562 \text{ lb}, B = 281 \text{ lb.}$$



Problem 10.79 The top of the rectangular plate is 2 m below the surface of a lake. Atmospheric pressure $p_{\text{atm}} = 1 \times 10^5 \text{ Pa}$ and the mass density of water is $\rho = 1000 \text{ kg/m}^3$.

- What is the maximum pressure exerted on the plate by the water?
- Determine the force exerted on a face of the plate by the pressure of the water.

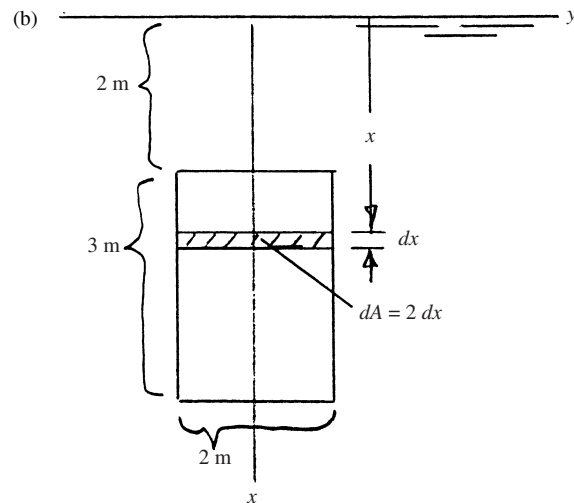
(See Example 10.11.)



Solution:

- The maximum pressure occurs at the bottom of the plate:

$$\begin{aligned} P &= P_0 + \gamma x \\ &= 1 \times 10^5 + (1000)(9.81)(5) \\ &= 1.49 \times 10^5 \text{ Pa.} \end{aligned}$$



The force is

$$\begin{aligned} F &= \int_A P dA \\ &= \int_2^5 (P_0 + \gamma x) 2 dx \\ &= \int_2^5 [1 \times 10^5 + (1000)(9.81)x] 2 dx \\ &= 806,000 \text{ N.} \end{aligned}$$

Problem 10.80 In Problem 10.79, how far below the top of the plate is the center of pressure located?

Solution: See the solution of Problem 10.79. The moment due to the pressure about the y axis is

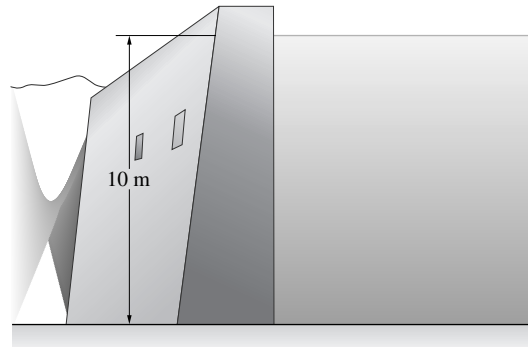
$$\begin{aligned} M &= \int_A x p dA \\ &= \int_2^5 x(P_0 + \gamma x)2 dx \\ &= \int_2^5 [1 \times 10^5 x + (1000)(9.81)x^2]2 dx \\ &= 2,865,000 \text{ N}\cdot\text{m}. \end{aligned}$$

The x -coordinate of the center of pressure is

$$x_p = \frac{M}{F} = \frac{2,865,000}{806,000} = 3.55 \text{ m}.$$

The distance below the top of the plate is $x_p - 2 \text{ m} = 1.55 \text{ m}$.

Problem 10.81 The width of the dam (the dimension into the page) is 100 m. The mass density of the water is $\rho = 1000 \text{ kg/m}^3$. Determine the force exerted on the dam by the gage pressure of the water (a) by integration; (b) by calculating the “volume” of the pressure distribution.



Solution: For gage pressure, we neglect the pressure of the atmosphere. (a) The weight density of the water is

$$\gamma = 1000(9.81) = 9810 \text{ N/m}^3.$$

The force on the dam is

$$F = \int_A p dA = 100\gamma \int_0^{10} x dx = 5000\gamma = 49.05 \times 10^6 \text{ N}$$

(b) The pressure distribution is a triangle, with base 10γ and altitude 10 m. The length dimension of this “solid” is 100 m. The volume of this triangular “solid” is

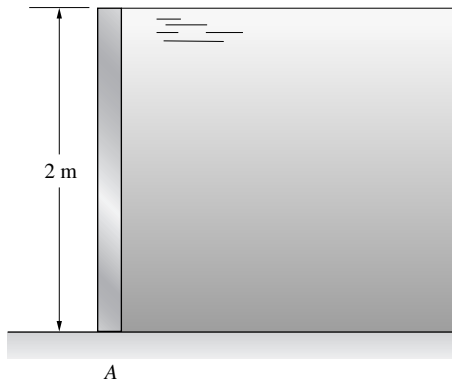
$$V = \left(\frac{1}{2}\right) 100(10)(9810) = 49.05 \times 10^6 \text{ N}$$

Problem 10.82 In Problem 10.81, how far down from the surface of the water is the center of pressure due to the gage pressure of the water on the dam?

Solution: The center of pressure is the centroid of the pressure distribution, which is a triangle of altitude 10 m and base 10γ . The centroid of a triangle is $\left(\frac{1}{3}\right)$ of the altitude, or, from the surface,

$$d = \left(\frac{2}{3}\right) 10 = 6.67 \text{ m}$$

Problem 10.83 The width of the gate (the dimension into the page) is 3 m. Atmospheric pressure $p_{\text{atm}} = 1 \times 10^5 \text{ Pa}$ and the mass density of the water is $\rho = 1000 \text{ kg/m}^3$. Determine the horizontal force and couple exerted on the gate by its built-in support A.

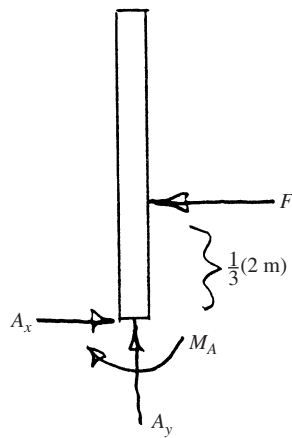
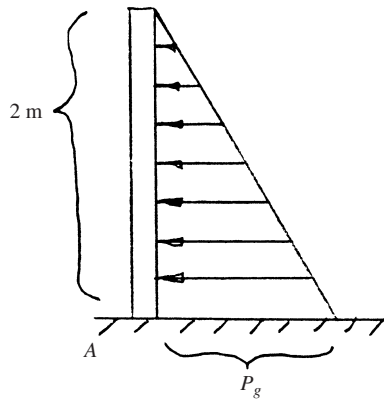


Solution: The gage pressure at the bottom of the gate is

$$P_g = \gamma x = (1000)(9.81)(2),$$

so the “volume” of the pressure distribution is

$$\begin{aligned} F &= \frac{1}{2}(2)P_g(3) \\ &= \frac{1}{2}(2)(1000)(9.81)(2)(3) \\ &= 58,860 \text{ N.} \end{aligned}$$

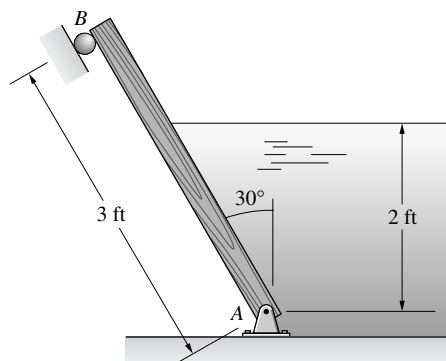


we see that

$$A_x = F = 58.9 \text{ kN,}$$

$$M_A = \frac{1}{3}(2)F = 39.2 \text{ kN-m.}$$

Problem 10.84 The homogenous gate weighs 100 lb, and its width (the dimension into the page) is 3 ft. The weight density of the water is $\gamma = 62.4 \text{ lb/ft}^3$, and the atmospheric pressure is $p_{\text{atm}} = 2120 \text{ lb/ft}^2$. Determine the reactions at A and B.



Solution: The atmospheric pressure acts on both sides of the gate, so it is ignored. The strategy is use the “volume” of the pressure distribution to compute the force acting on the face of the gate. The pressure distribution is a triangle with base 2γ . The pressure acts over an area

$$\left(\frac{6}{\cos(30^\circ)} \right) = 6.92 \text{ ft}^2.$$

Thus the “volume” is $F = \left(\frac{1}{2} \right) (62.4)(2)(6.92) = 432.32 \text{ lb}$. This force acts normally to the surface of the gate, or at an angle of $\theta = 210^\circ$ relative to the positive x axis. The centroid of the pressure is

$$d = \left(\frac{2}{3} \right) \left(\frac{1}{\cos 30^\circ} \right) = 0.7698 \text{ ft}$$

along the inner face of the gate from A. The sum of the moments about A is

$$\sum M_A = dF + W(1.5) \sin 30^\circ - 3B = 0,$$

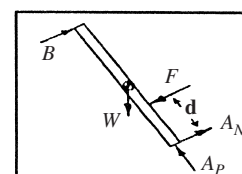
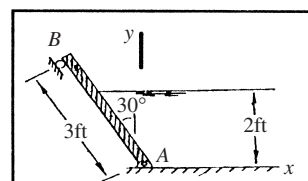
from which $B = 135.933 \text{ lb}$, normal to the inner face of the gate. The sum of the forces normal to the gate surface is

$$\sum F_N = +A_N - F + B - W \sin 30^\circ = 0,$$

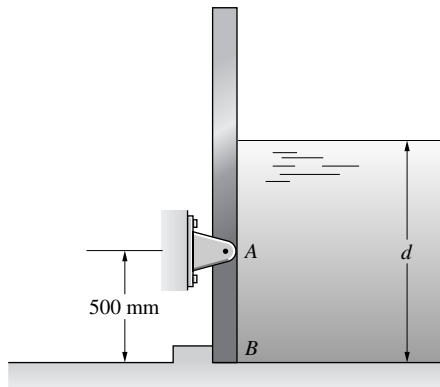
from which $A_N = 346.4 \text{ lb}$ at an angle $\alpha = 30^\circ$ relative to the positive x axis. The sum of the forces acting parallel to the gate surface is

$$\sum F_P = A_P - W \cos(30^\circ) = 0,$$

from which $A_P = 86.6 \text{ lb}$ at an angle of 120° . Thus the components of the reaction at A are $A_x = -346.3 \cos(210^\circ) + 86.6 \cos(120^\circ) = 256.7 \text{ lb}$, to the right, and $A_y = -346.3 \sin(210^\circ) + 86.6 \sin(120^\circ) = 248.2 \text{ lb}$ upward.

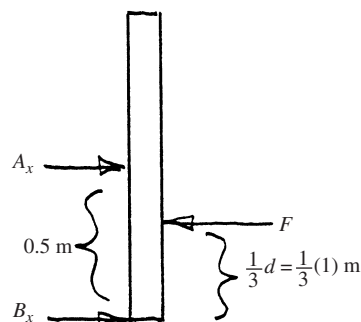
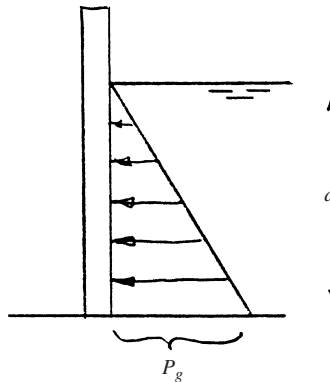


Problem 10.85 The width of the gate (the dimension into the page) is 2 m and there is water of depth $d = 1$ m on one side. Atmospheric pressure $p_{\text{atm}} = 1 \times 10^5$ Pa and the mass density of the water is $\rho = 1000$ kg/m³. Determine the horizontal forces exerted on the gate at A and B .



Solution: The “volume” of the gage pressure distribution is

$$\begin{aligned} F &= \frac{1}{2} \rho g d (2) \\ &= \delta p d^2 \\ &= (1000)(9.81)(1)^2 \\ &= 9810 \text{ N.} \end{aligned}$$



Applying the equilibrium equations, we find that

$$A_x = \frac{2}{3} F = 6540 \text{ N,}$$

$$B_x = \frac{1}{3} F = 3270 \text{ N.}$$

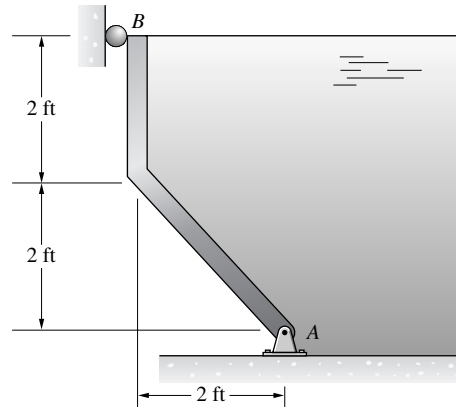
Problem 10.86 The gate in Problem 10.85 is designed to rotate and release the water when the depth d exceeds a certain value. What is that depth?

Solution: See the solution of Problem 10.85. The gate rotates when

$$\frac{1}{3} d > \frac{1}{2} \text{ m,}$$

$$d > \frac{3}{2} \text{ m.}$$

Problem 10.87* The dam has water of depth 4 ft on one side. The width of the dam (the dimension into the page) is 8 ft. The weight density of the water is $\gamma = 62.4 \text{ lb/ft}^3$, and the atmospheric pressure $p_{\text{atm}} = 2120 \text{ lb/ft}^2$. If you neglect the weight of the dam, what are the reactions at A and B?



Solution: To simplify the analysis of the pressure forces, we will draw a free body diagram of the dam and the volume of water shown: The left side and top of the free body diagram are subjected to atmospheric pressure, and the right side is subjected to the sum of atmospheric pressure and the gage pressure of the water, so we only need to consider the gage pressure. Let us represent the pressure force on the right side by an equivalent force: We can determine F by calculating the “volume” of the pressure distribution:

$$F = \frac{1}{2} p(4 \text{ ft})(8 \text{ ft}) = \frac{1}{2} \gamma(4)(4)(8) = 3990 \text{ lb}$$

the complete free body diagram is: The two weights are

$$w_1 = \gamma(2 \text{ ft})(2 \text{ ft})(8 \text{ ft}) = 2000 \text{ lb}$$

$$w_2 = \frac{1}{2} \gamma(2 \text{ ft})(2 \text{ ft})(8 \text{ ft}) = 988 \text{ lb.}$$

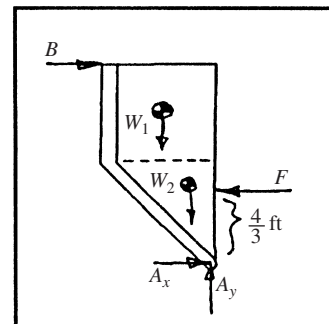
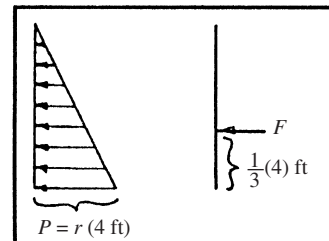
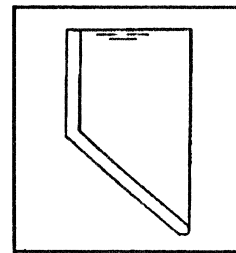
From the equilibrium equations

$$\sum F_x = A_x + B - F = 0$$

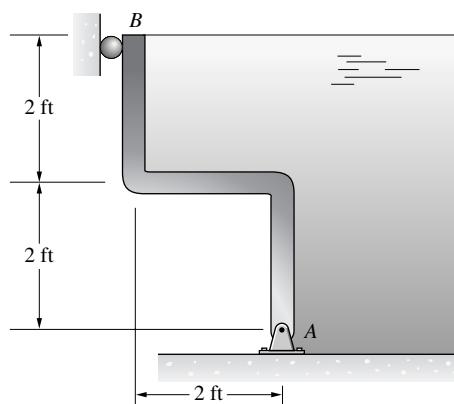
$$\sum F_y = A_y - w_1 - w_2 = 0$$

$$\sum M_{(pt.A)} = -4B + \left(\frac{4}{3}\right) F + (1)W_1 + \frac{1}{3}W_2 = 0,$$

we obtain $A_x = 2000 \text{ lb}$, $A_y = 3000 \text{ lb}$, $B = 2000 \text{ lb}$.



Problem 10.88* The dam has water of depth 4 ft on one side. The width of the dam (the dimension into the page) is 8 ft. The weight density of the water is $\gamma = 62.4 \text{ lb/ft}^3$, and atmospheric pressure is $p_{\text{atm}} = 2120 \text{ lb/ft}^2$. If you neglect the weight of the dam, what are the reactions at A and B?



Solution: The atmospheric pressure acts on both faces of the dam, so it is ignored. The strategy is to use the “volume” of the pressure distribution to determine the reactions. The pressure distribution is a triangle of base 4γ and altitude 4 ft. The force on the vertical faces of the dam is

$$F_1 = \left(\frac{1}{2}\right)(4)(4)(\gamma)8 = 3993.6 \text{ lb.}$$

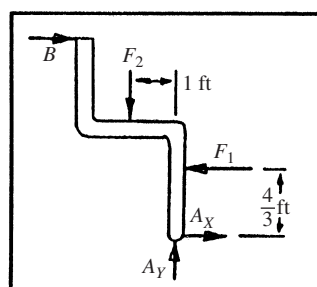
The moment about A due to the force on the vertical faces is

$$M_1 = \left(\frac{4}{3}\right)F_1 = 5324.8 \text{ ft lb.}$$

The force on the horizontal face of the dam is

$$F_2 = (2)(\gamma)(2)(8) = 1996.8 \text{ lb.}$$

The moment about A due to the force on the horizontal face is $M_2 = 1F_2 = 1996.8 \text{ ft lb}$. The sum of the moments about A: $\sum M_A = M_1 + M_2 - 4B = 0$, from which $B = 1830.4 \text{ lb}$. The sum of the forces: $\sum F_x = A_x - F_1 + B = 0$, from which $A_x = 2163.3 \text{ lb}$ to the right. $\sum F_y = A_y - F_2 = 0$, from which $A_y = 1996.8 \text{ lb}$ upward.

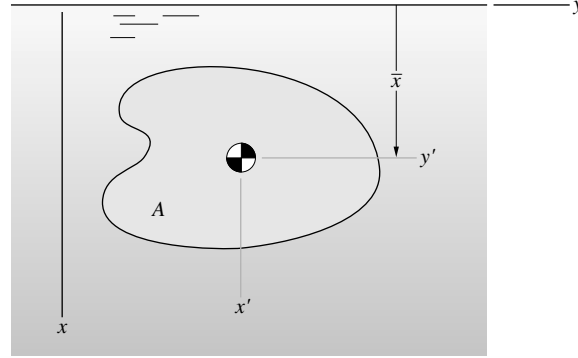


Problem 10.89 Consider a plane, vertical area A below the surface of a liquid. Let p_0 be the pressure at the surface.

- Show that the force exerted on the area is $F = \mathbf{p}A$, where $\mathbf{p} = p_0 + \gamma \mathbf{x}$ is the pressure of the liquid at the centroid of the area.
- Show that the x coordinate of the center of pressure is

$$x_P = \mathbf{x} + \frac{\gamma I_{y'}}{\mathbf{p}A},$$

where $I_{y'}$ is the moment of inertia of the area about the y' axis through its centroid.



Solution: (a) The definition of the centroid of the area is

$$\mathbf{x}A = \int_A x dA.$$

The force on the plate is

$$F = \int_A p dA = p_0 \int_A dA + \gamma \int_A x dA = p_0 A + \gamma \mathbf{x}A.$$

But by definition, the pressure at the centroid of the area is $\mathbf{p} = p_0 + \gamma \mathbf{x}$, hence $F = \mathbf{p}A$.

(b) The moment about the y axis is

$$M_{y\text{-axis}} = \int_A px dA = p_0 \int_A x dA + \gamma \int_A x^2 dA.$$

The moment of inertia is defined:

$$I_y A = \int_A x^2 dA,$$

from which the moment is $M_{y\text{-axis}} = p_0 \mathbf{x}A + \gamma I_y$. But the moment is also given by $M_{y\text{-axis}} = F x_P$, where F is the force on the plate and x_P is the distance to the center of pressure. Thus $\mathbf{p}A x_P = p_0 \mathbf{x}A + \gamma I_y$,

$$\text{from which } x_P = \left(\frac{p_0}{\mathbf{p}} \right) \mathbf{x} + \frac{\gamma I_y}{\mathbf{p}A}.$$

From the parallel axis theorem, $I_y = \mathbf{x}^2 A + I_{y'}$. Substitute and reduce:

$$x_P = \frac{(p_0 + \gamma \mathbf{x}) \mathbf{x}}{\mathbf{p}} + \frac{\gamma I_{y'}}{\mathbf{p}A} = \mathbf{x} + \frac{\gamma I_{y'}}{\mathbf{p}A},$$

which demonstrates the required result.

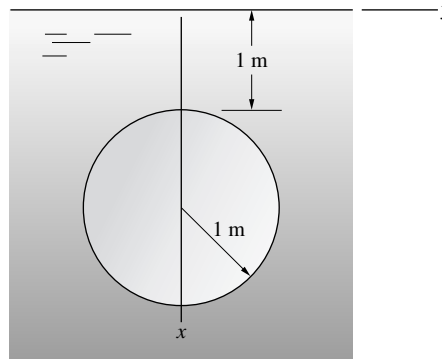
Problem 10.90 A circular plate of 1-m radius is below the surface of a stationary pool of water. Atmospheric pressure is $p_{\text{atm}} = 10^5 \text{ Pa}$, and the mass density of the water is $\rho = 1000 \text{ kg/m}^3$. Determine (a) the force exerted on the face of the plate by the pressure of the water; (b) the x coordinate of the center of pressure. (See Problem 10.89.)

Solution: (a) From Problem 10.89, the pressure on the face of the plate is $F = \mathbf{p}A$, where $\mathbf{p}$ is the pressure at the centroid of the area.

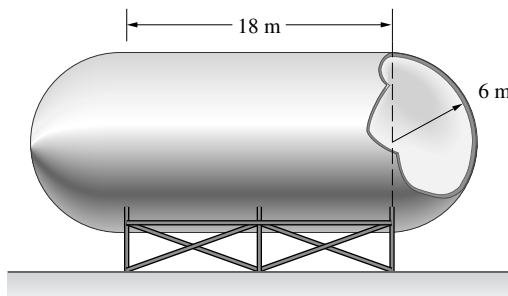
$$F = (p_0 + \gamma y)(\pi 1^2) = 375.8 \text{ kN.}$$

(b) From Problem 10.99,

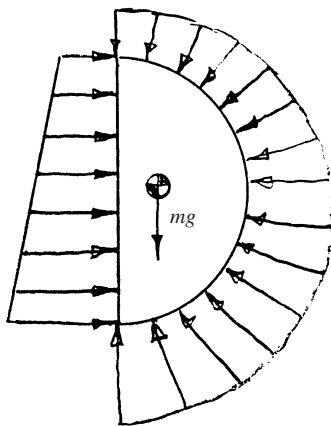
$$x_P = x + \frac{\gamma I_{y'}}{\mathbf{p}A} = 2 + \frac{9810 \left(\frac{\pi 1^4}{4} \right)}{F} = 2.0205 \text{ m}$$



Problem 10.91* A tank consists of a cylinder with hemispherical ends. It is filled with water ($\rho = 1000 \text{ kg/m}^3$). The pressure of the water at the top of the tank is 140 kPa. Determine the magnitude of the force exerted by the pressure of the water on each hemispherical end of the tank. (See Example 10.12.)



Solution: The free-body diagram is



The force exerted by the pressure distribution on the curved surface (equal and opposite to the force exerted on the tank's hemispherical end) can be determined from the fact that the free-body diagram is in equilibrium.

The magnitude of the vertical component is

$$mg = (1000) \frac{1}{2} \left(\frac{4}{3} \pi R^3 \right) (9.81) = 4.44 \times 10^6 \text{ N.}$$

To determine the horizontal component, see Problem 10.99. The pressure at the centroid is

$$\mathbf{p} = 140,000 + (1000)(9.81)(6) = 199,000 \text{ Pa,}$$

so the horizontal component is

$$\mathbf{p}A = (199,000)\pi(6)^2 = 22.5 \times 10^6 \text{ N.}$$

The magnitude of the force is

$$\sqrt{(mg)^2 + (\mathbf{p}A)^2} = 22.9 \text{ MN.}$$

Problem 10.92 An object of volume V and weight W is suspended below the surface of the stationary liquid of weight density γ (Fig. a). Show that the tension in the cord is $W - V\gamma$. In other words, show that the pressure distribution on the surface of the object exerts an upward force equal to the product of the object's volume and the weight density of the water. The result is due to Archimedes (287–212 B.C.).

Strategy: Draw the free-body diagram of a volume of liquid that has the same shape and position as the object (Fig. b).

Solution: The result follows from the free body diagram of the space occupied by the object and the development of the force exerted by the pressure in terms of the volume of the object. Let A be the area bounding the volume V . The force exerted on a surface A is

$$\mathbf{F} = - \int_A \mathbf{n} p dA,$$

where $\mathbf{n}$ is a unit vector normal to the elemental surface dA , positive outward from the surface, and the negative sign comes from the equilibrium condition that the reaction force acts oppositely to the unit vector. Choose an x, y, z coordinate system such that the elemental forces are

$$\mathbf{n} p dA = \mathbf{i} p dy dz + \mathbf{j} p dx dz + \mathbf{k} p dx dy.$$

The force becomes

$$\mathbf{F} = - \int_A \mathbf{i} p dy dz - \int_A \mathbf{j} p dx dz - \int_A \mathbf{k} p dx dy.$$

But these integrals can also be written as

$$\mathbf{F} = -\mathbf{i} \int_V \frac{\partial p}{\partial x} dx dy dz - \mathbf{j} \int_V \frac{\partial p}{\partial y} dx dy dz - \mathbf{k} \int_V \frac{\partial p}{\partial z} dx dy dz.$$

This set of integrals can be collapsed into

$$\mathbf{F} = - \int_A \mathbf{n} p dA = - \int_V \nabla p dV,$$

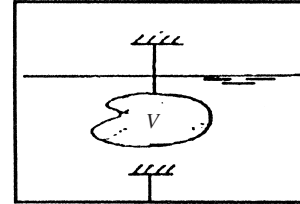
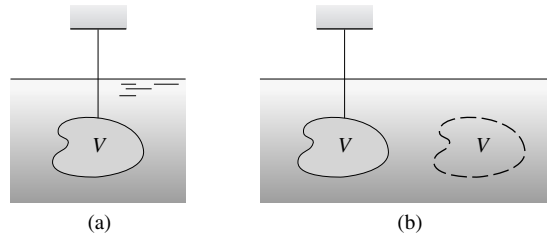
where the volume V is bounded by the surface A , ∇p is a shorthand notation for

$$\nabla p = \mathbf{i} \frac{\partial p}{\partial x} + \mathbf{j} \frac{\partial p}{\partial y} + \mathbf{k} \frac{\partial p}{\partial z},$$

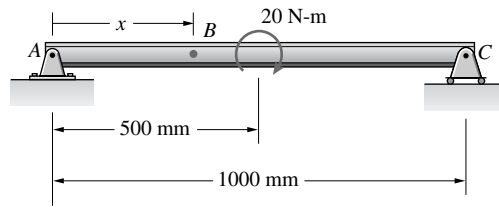
and $dV = dx dy dz$. The pressure $p = p_0 + \gamma x$, from which $\nabla p = \mathbf{i} \gamma$, and the integral becomes

$$\mathbf{F} = - \int_A \mathbf{n} p dA = -\mathbf{i} \gamma \int_V dV = -\mathbf{i} \gamma V.$$

The weight of the object acts in the positive x direction, so the resultant force is $\mathbf{F}_R = \mathbf{i}(W - \gamma V)$



Problem 10.93 Determine the internal forces and moment at B (a) if $x = 250$ mm; (b) if $x = 750$ mm.



Solution: (a) The sum of the moments about A

$$\sum M_A = -20 + 1C = 0,$$

from which $C = 20$ N. The sum of forces

$$\sum F_y = A_y + C = 0,$$

from which $A_y = -C = -20$ N.

$$\sum F_x = A_x = 0$$

(a) *Make a cut at B.* Isolate the left hand part. The sum of moments

$$\sum M = M_B + 0.25(20) = 0,$$

from which

$$M_B = -5 \text{ N}\cdot\text{m}.$$

$$V_B = -20 \text{ N},$$

$$P_B = 0.$$

(b) *Make a cut at B:* Isolate the left hand part. The sum of moments:

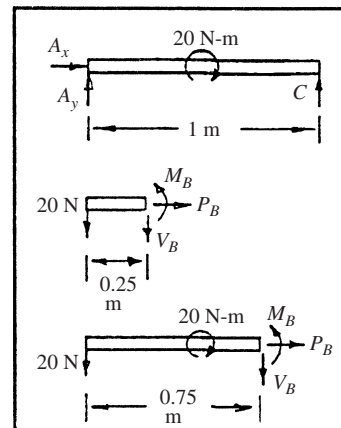
$$\sum M = M_B - 20 + 0.75(20) = 0,$$

from which

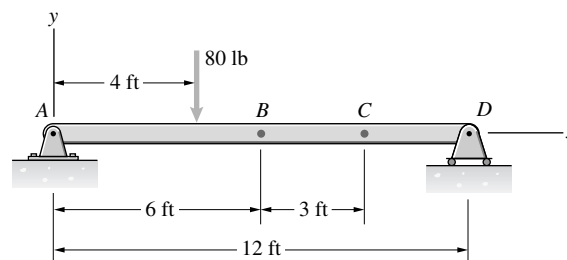
$$M_B = 5 \text{ N}\cdot\text{m}.$$

$$V_B = -20 \text{ N},$$

$$P_B = 0.$$



Problem 10.94 Determine the internal forces and moment (a) at B; (b) at C.



Solution: The sum of the moments about A is

$$\sum M_A = -4(80) + 12D = 0,$$

from which $D = \frac{80}{3}$ lb. The sum of forces:

$$\sum F_y = A_y - 80 + D = 0,$$

from which $A_y = \frac{160}{3} = 53.33$ lb.

$$\sum F_x = A_x = 0$$

(a) Make a cut at B. Isolate the left hand part. The sum of moments

$$\sum M = M_B + 2(80) - 6(53.33) = 0,$$

from which

$$M_B = 160 \text{ ft lb.}$$

$$V_B = 53.33 - 80 = -26.7 \text{ lb.}$$

$$P_B = 0$$

(b) Make a cut at C: The sum of moments

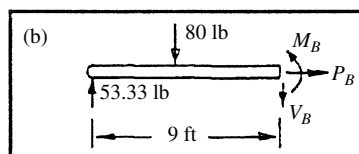
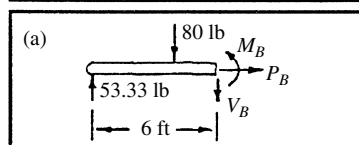
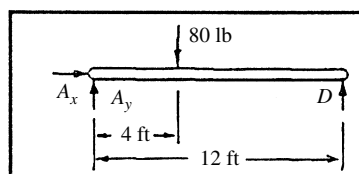
$$\sum M = M_C + 80(5) - 9(53.33) = 0,$$

from which

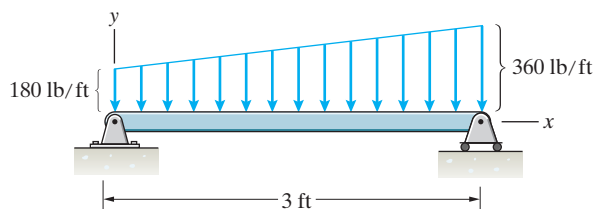
$$M_C = 80 \text{ ft lb.}$$

$$V_C = -80 + 53.33 = -26.7 \text{ lb.}$$

$$P_C = 0$$



Problem 10.95 (a) Determine the maximum bending moment in the beam and the value of x where it occurs. (b) Show that the equations for V and M as functions of x satisfy the equation $V = dM/dx$.



Solution: Find the reactions first

$$\sum M_A: -A(3 \text{ ft}) + (540 \text{ lb})(1.5 \text{ ft}) + (270 \text{ lb})(1.0 \text{ ft}) = 0$$

$$\sum F_y: A + B - 540 \text{ lb} - 270 \text{ lb} = 0$$

$$A = 360 \text{ lb}, \quad B = 450 \text{ lb}$$

$$w = 180 \text{ lb/ft} + \frac{180 \text{ lb/ft}}{3 \text{ ft}}x = 180 \text{ lb/ft} + (60 \text{ lb/ft}^2)x$$

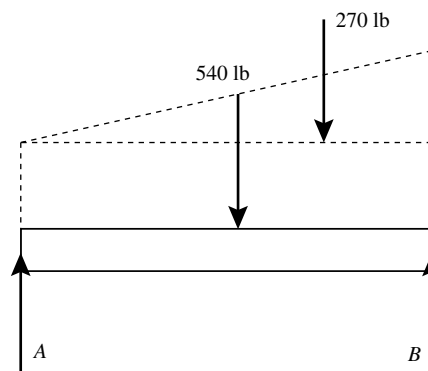
$$V = -(180 \text{ lb/ft})x - (30 \text{ lb/ft}^2)x^2 + 360 \text{ lb}$$

$$M = -(90 \text{ lb/ft})x^2 - (10 \text{ lb/ft}^2)x^3 + (360 \text{ lb})x$$

(a) The maximum moment occurs when the shear force is zero

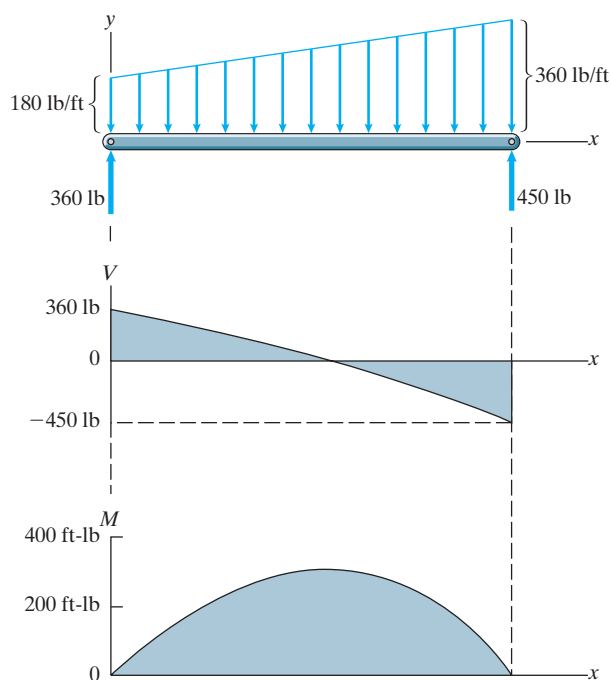
$$V = 0 \Rightarrow x = 1.583 \text{ ft} \Rightarrow M = 305 \text{ ft}\cdot\text{lb}$$

$$(b) \quad \frac{dM}{dx} = -(180 \text{ lb/ft})x - (30 \text{ lb/ft}^2)x^2 + 360 \text{ lb} = V$$

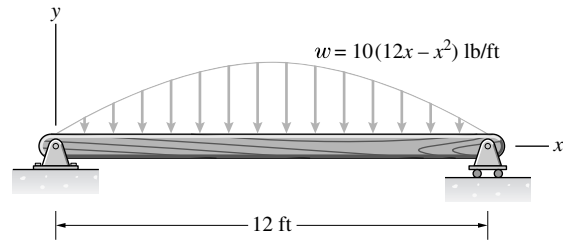


Problem 10.96 Draw the shear force and bending moment diagrams for the beam in Problem 10.95.

Solution: Plot the solution of 10.95



Problem 10.97 Determine the shear force and bending moment diagram's for the beam.



Solution: Denote the reactions at the left and right ends by A and B , respectively. The total force due to the load is

$$F = \int_0^{12} 10(12x - x^2) dx = 10 \left[6x^2 - \frac{x^3}{3} \right]_0^{12} = 2880 \text{ lb.}$$

The moment about the left end due to the load is

$$M_{\text{LOAD}} = \int_0^{12} 10(12x - x^2)x dx = 10 \left[4x^3 - \frac{x^4}{4} \right]_0^{12}.$$

$$M_{\text{LOAD}} = 17280 \text{ ft lb.}$$

The sum of the moments about the left end:

$$\sum M_A = -M + 12B = 0,$$

from which $B = 1440$ lb. The sum of the forces:

$$\sum F_y = A_y + B - F = 0,$$

from which $A_y = 1440$ lb. Beginning from the left end, the shear is

$$V(x) = - \int_0^x 10(12x - x^2) + 1440$$

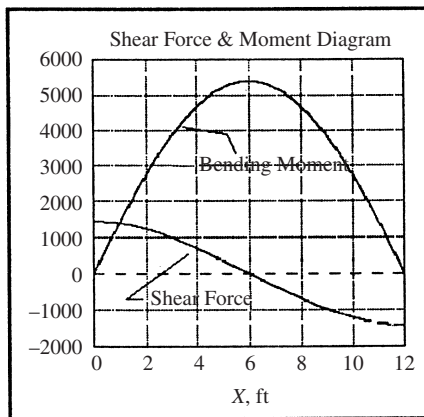
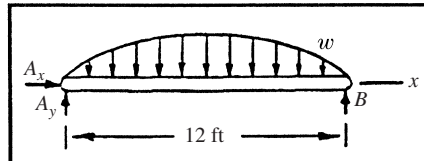
$$V(x) = -10 \left(6x^2 - \frac{x^3}{3} \right) + 1440.$$

The moment is

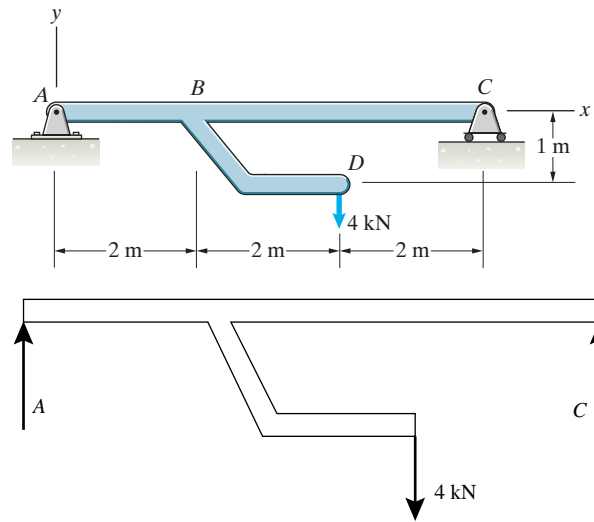
$$M = \int V(x) dx + C = -10 \left(2x^3 - \frac{x^4}{12} \right) + 1440x + C.$$

The moment is zero at $x = 0$, hence the constant $C = 0$, and the moment is

$$M = -10 \left(2x^3 - \frac{x^4}{12} \right) + 1440x$$



Problem 10.98 Determine V and M as functions of x for the beam ABC .



Solution: First find the reactions

$$\sum M_C : -A(6 \text{ m}) + (4 \text{ kN})(2 \text{ m}) = 0$$

$$\sum F_y : A + C - 4 \text{ kN} = 0$$

$$A = 1.33 \text{ kN}, \quad C = 2.67 \text{ kN}$$

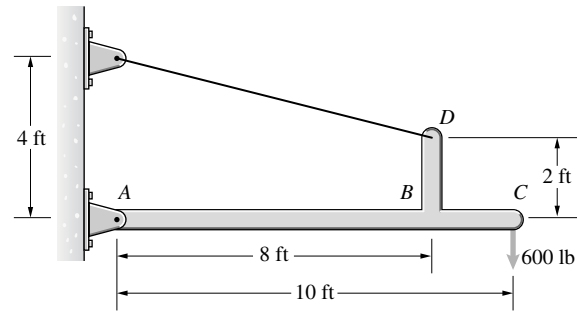
In the first region $0 < x < 2 \text{ m}$

$$\begin{aligned} w_1 &= 0 \\ V_1 &= 1.33 \text{ kN} \\ M_1 &= (1.33 \text{ kN})x \end{aligned}$$

In the second region $2 \text{ m} < x < 6 \text{ m}$

$$\begin{aligned} w_2 &= 0 \\ V_2 &= -2.67 \text{ kN} \\ M_2 &= -(2.67 \text{ kN})x + 16 \text{ kN-m} \end{aligned}$$

Problem 10.99 Draw the shear force and bending moment diagrams for beam *ABC*.



Solution: *The structure as a free body:* The angle of the cable at *D* relative to the horizontal is

$$\theta = \tan^{-1} \left(\frac{1}{4} \right) = 14^\circ.$$

Denote the tension in the cable by *T*. The sum of the moments about *A* is

$$\sum M_A = 2T \cos \theta + 8T \sin \theta - 10(600) = 0,$$

from which $T = 1546.2$ lb. The sum of the forces:

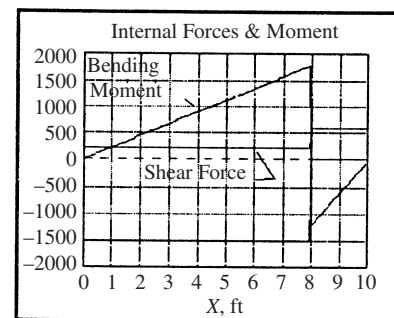
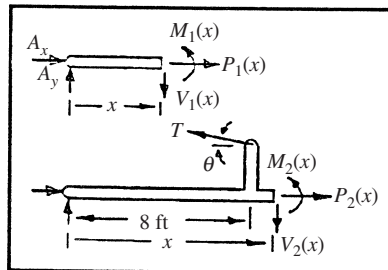
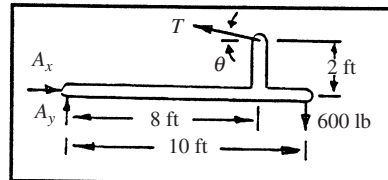
$$\sum F_y = A_y - 600 + T \sin \theta = 0,$$

from which $A_y = 225$ lb. *The intervals as free bodies:* Divide the beam into two intervals: $0 \leq x < 8$, and $8 \leq x \leq 10$. *Interval 1:* The shear force is $V_1(x) = A_y$. The internal bending moment is $M_1(x) = A_y x = 225x$. *Interval 2:* The shear force is $V_2(x) = A_y + T \sin \theta = 600$ lb. The sum of the moments is

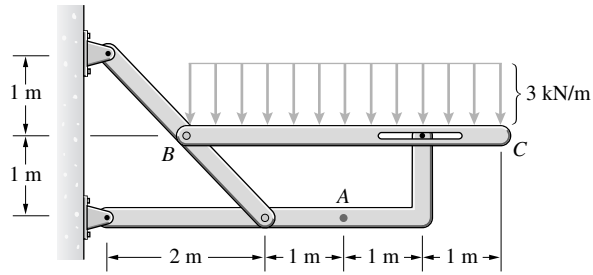
$$\sum M^x(x) = M_2(x) - A_y x - (T \sin \theta)(x - 8) + 2T \cos \theta = 0,$$

from which $M_2(x) = 600x - 6000$ ft lb.

The diagrams are shown. The discontinuity in the moment at $x = 8$ is due to the moment exerted on the beam by the horizontal component of the cable tension at *D*. This horizontal component of tension exerts a moment that is independent of the distance x along the beam *ABC*. In this sense, it behaves like a couple.



Problem 10.100 Determine the internal forces and moments at A.



Solution: Denote the reactions at the slide support of BC by D , and the reactions at the upper and lower pin supports by E and G respectively, and the reaction at the pin connection on the beam GB by H . The total force due to the load is

$$F = \int_0^4 3 dx = 12 \text{ kN.}$$

The moment about the left end of BC is

$$M = \int_0^4 3x dx = 24 \text{ kN m.}$$

The sum of the moments about the point B is

$$\sum M_B = -M + 3D = 0,$$

from which $D = 8 \text{ kN}$. The sum of the forces:

$$\sum F_y = B - F + D = 0,$$

from which $B = 4 \text{ kN}$ is the vertical reaction at B . The centroid distance of the load from the left end of BC is

$$d = \frac{M}{F} = \frac{24}{12} = 2 \text{ m.}$$

The sum of the moments about the upper pin support is

$$\sum M = -(1 + 2)F + 2G_x = 0,$$

from which $G_x = 18 \text{ kN}$. With these known quantities, the following equations can be written: (See figure to right for assumed positive directions.)

Sum of moments about H for lower member yields $G_y = -8 \text{ kN}$. The sum of the forces for the lower member yields:

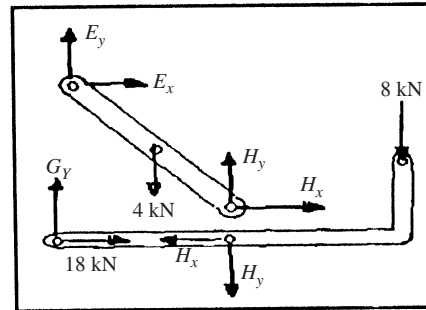
$$H_y = -16 \text{ kN}$$

and $H_x = 18 \text{ kN}$.

Divide the lower member into two parts

$$0 \leq x < 2 \text{ (m)}$$

and $2 \leq x < 3 \text{ (m)}$.



Part 1: The shear is $V_1(x) = -8 \text{ kN}$. The moment is

$$M_1(x) = \int V_1(x) dx + C_1 = -8x + C_1.$$

The moment at $x = 0$, $M_1(0) = 0$, from which $C_1 = 0$, and $M_1(x) = -8x$. *Part 2:* The shear is $V_2(x) = V_1(x) + 16 = 8 \text{ kN}$. The moment is

$$M_2(x) = \int V_2(x) dx + C_2 = 8x + C_2.$$

The moment is continuous at $x = 2$, $M_1(2) = M_2(2)$, from which $C_2 = -32$, and the moment is $M_2(x) = 8x - 32 \text{ kN m}$. Thus at point A, $x = 3$, the internal forces and bending moment are

$$V_2(3) = 8 \text{ kN,}$$

$$P(3) = 0,$$

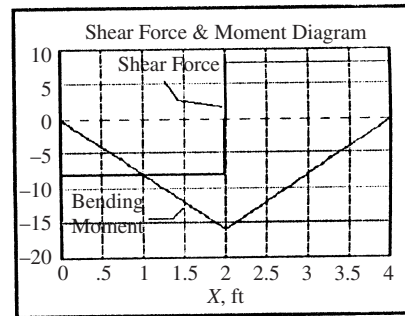
$$M_2(3) = 24 - 32 = -8 \text{ kN m}$$

Problem 10.101 Draw the shear force and bending moment diagrams of beam BC in Problem 10.100.

Solution: From the solution to Problem 10.101, the shear force and bending moment are:

- (1) $V_1(x) = -8 \text{ kN}$, $M_1(x) = -8x \text{ kN m}$, ($0 \leq x < 2 \text{ m}$),
- (2) $V_2(x) = 8 \text{ kN}$, $M_2(x) = 8x - 32 \text{ kN m}$, ($2 \leq x < 4 \text{ m}$)

The shear force and bending moment diagrams are shown.



Problem 10.102 Determine the internal forces and moments at B (a) if $x = 250 \text{ mm}$; (b) if $x = 750 \text{ mm}$.

Solution: *The complete beam:* The sum of the moments about A is

$$\sum M_A = M_A - 20 + 1(40) = 0,$$

from which $M_A = -20 \text{ N m}$. The sum of the forces:

$$\sum F_y = A_y + 40 = 0,$$

from which $A_y = -40 \text{ N}$.

$$\sum F_x = A_x = 0$$

The internal forces at $x = 250 \text{ mm}$. The shear is $V_1(x) = -40 \text{ N}$. The moment is

$$M_1(x) = \int V_1(x) dx + C_1 = -40x + C_1$$

At $x = 0$, $M_1(0) = -M_A = 20 \text{ N m}$, from which $C_1 = 20 \text{ kN}$. Thus the moment is $M(0.25) = -40(0.25) + 20 = 10 \text{ Nm}$. *The internal forces at $x = 750 \text{ mm}$.* Divide the beam into two segments:

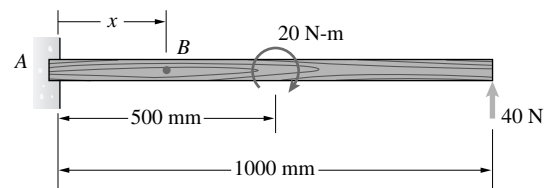
$$0 \leq x < 0.5 \text{ (m)},$$

$$\text{and } 0.5 \leq x < 0.75 \text{ (m)}.$$

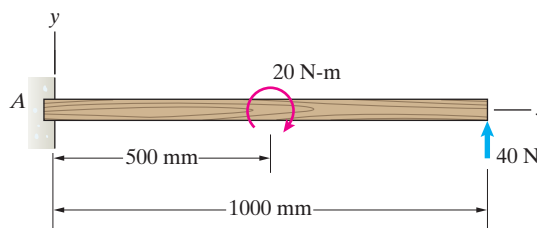
The shear in the second segment is $V_2(x) = -40 \text{ N}$. The moment is

$$M_2(x) = \int V_2(x) dx + C_2 = -40x + C_2.$$

A known discontinuity exists in the moment at $x = 0.5 \text{ m}$, $M_1(0.5) - M_2(0.5) = -20 \text{ N m}$, from which $C_2 = 40$ and the moment is $M_2(x) = -40x + 40 \text{ Nm}$. At $x = 0.75$, $M_2(0.75) = -40(0.75) + 40 = 10 \text{ N m}$. The axial forces $P(x) = 0$ everywhere.



Problem 10.103 Draw the shear force and bending moment diagrams.



Solution: From the solution to Problem 10.102, the shear and bending moment are

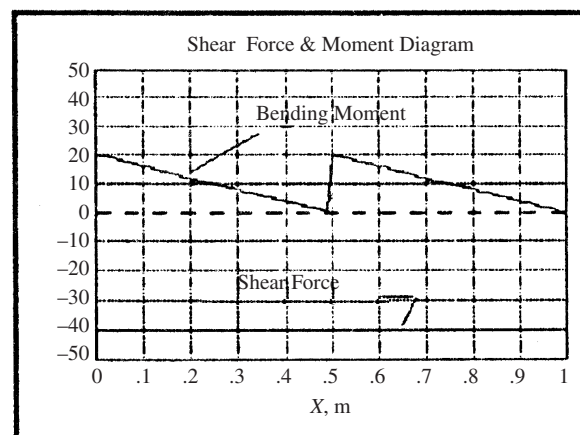
$$V_1(x) = -40 \text{ N},$$

$$M_1(x) = -40x + 20 \text{ Nm} \quad (0 \leq x < 0.5 \text{ m}),$$

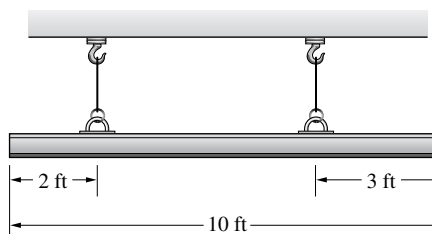
and $V_2(x) = -40 \text{ N}$,

$$M_2(x) = -40x + 40 \text{ Nm}, \quad (0.5 \leq x < 1 \text{ m})$$

The shear force and bending moment diagrams are shown.



Problem 10.104 The homogenous beam weighs 1000 lb. What are the internal forces and bending moments at its midpoint?



Solution: Denote the support reactions by A and B . The load distribution is $w = 1000/10 = 100 \text{ lb/ft}$. The moment about the left end due to the load of the beam is

$$M_L = \int_0^{10} wx \, dx = 100 \left[\frac{x^2}{2} \right]_0^{10} = 5000 \text{ ft lb}.$$

The sum of the moments about the left end is

$$\sum M = -M_L + 2A + 7B = 0$$

The sum of the forces:

$$\sum F_y = A + B - 1000 = 0.$$

Solve the two simultaneous equations to obtain: $A = 400 \text{ lb}$, $B = 600 \text{ lb}$.

$$\sum F_x = 0.$$

Divide the left half of the beam into two parts: $0 \leq x < 2 \text{ (ft)}$ and $2 \leq x < 5 \text{ (ft)}$.

Part 1: The shear is $V_1(x) = -100x$. The moment is

$$M_1(x) = \int V_1(x) \, dx + C_1 = -50x^2 + C_1.$$

At $x = 0$, $M_1(0) = 0$, thus $C_1 = 0$, and the moment is $M_1(x) = -50x^2$.

Part 2: The shear is $V_2(x) = V_1(x) + 400 \text{ lb}$. The moment is

$$M_2(x) = \int V_2(x) \, dx + C_2 = -50x^2 + 400x + C_2.$$

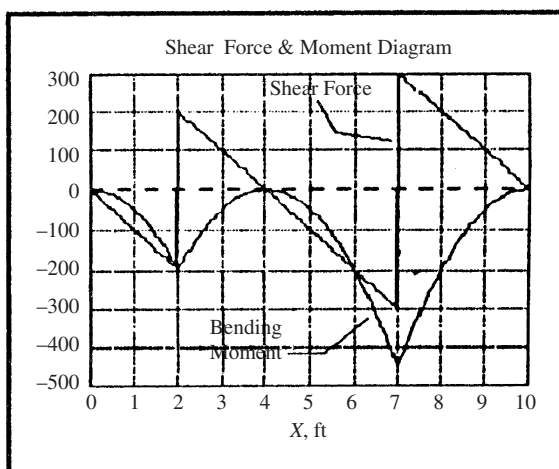
The moment is continuous at $x = 2$, $M_1(2) = M_2(2)$, from which $C_2 = -800$, and the moment is $M_2(x) = -50x^2 + 400x - 800$. At the midpoint, $x = 5 \text{ ft}$, the shear is $V_2(5) = -100(5) + 400 = -100 \text{ lb}$ and the moment is $M_2(5) = -50(25) + 400(5) - 800 = -50 \text{ ft lb}$. The axial force is zero, $P(5) = 0$.

Problem 10.105 Draw the shear force and bending moment diagrams for the beam in Problem 10.104.

Solution: From the solution for Problem 10.104, the shear force and bending moment are:

- (1) $V_1(x) = -100x$ lb, $M_1(x) = -50x^2$ ft lb, ($0 \leq x < 2$ ft).
- (2) $V_2(x) = -100x + 400$ lb, $M_2(x) = -50x^2 + 400x - 800$ ft lb, ($2 \leq x < 7$ ft).
- (3) $V_3(x) = -100x + 1000$ lb, $M_3(x) = -50x^2 + 1000x - 5000$ ft lb, ($7 \leq x < 10$ ft).

The shear force and bending moment diagrams are shown.



Problem 10.106 At A the main cable of the suspension bridge is horizontal and its tension is 1×10^8 lb.

- (a) Determine the distributed load acting on the cable.
- (b) What is the tension at B ?

Solution: (a) The parameter

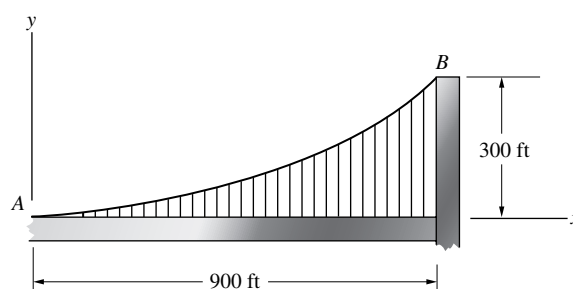
$$a = 2 \frac{y}{x^2} = 2 \frac{300}{900^2} = 7.4074 \times 10^{-4}.$$

The distributed load is

$$w = T_0 a = (1 \times 10^8)(7.4 \times 10^{-4}) = 7.4074 \times 10^4 \text{ lb/ft}$$

(b) The tension at B is

$$T_B = T_0 \sqrt{1 + a^2(900)^2} = 1.2 \times 10^8 \text{ lb}$$



Problem 10.107 The power line has a mass of 1.4 kg/m. If the line will safely support a tension of 5 kN, determine whether it will safely support an ice accumulation of 4 kg/m.

Solution: The power line meets the conditions for a catenary. The weight density with an ice load is

$$\gamma = (1.4 + 4)(9.81) = 52.974 \text{ N/m}.$$

The angle at the attachment point is related to the length and the parameter a by $sa = \tan \theta$. But $sa = \sinh(ax)$, and x is known. Thus the parameter can be found from

$$a = \frac{\sinh^{-1}(\tan \theta)}{x} = \frac{\sinh^{-1}(0.2126)}{20} = 0.01055.$$

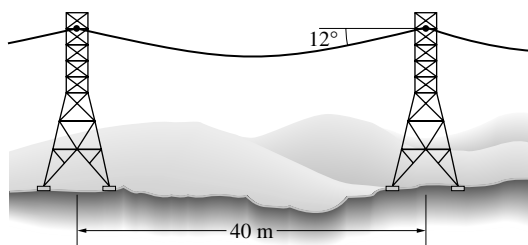
The tension at the lowest point is

$$T_0 = \frac{w}{a} = 5021.53 = 5.02 \text{ kN}.$$

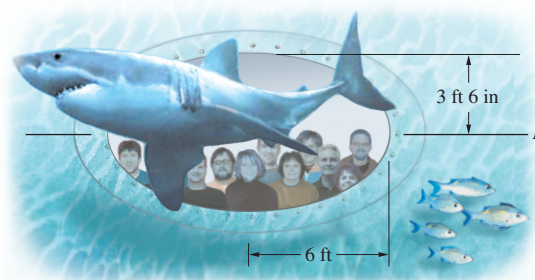
The maximum tension is

$$T = T_0 \cosh(ax) = 5133.7 = 5.133 \text{ kN}.$$

Thus the line will not sustain the load.



Problem 10.108 The water depth at the center of the elliptical aquarium window is 20 ft. Determine the magnitude of the net force exerted on the window by the pressure of the seawater ($\gamma = 64 \text{ lb/ft}^3$) and the atmospheric pressure of the air on the opposite side. (See Problem 10.89.)



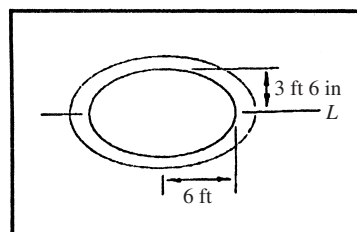
Solution: The force on the plate is

$$F = \int_A p \, dA.$$

The pressure is $p = p_0 + \gamma x - p_0 = \gamma x$, where the atmospheric pressure cancels, since it appears on both sides. The force:

$$F = \gamma \int_A x \, dA = \gamma \bar{x} A,$$

hence the force on the window is $F = 1280A \text{ lb}$. The area of the ellipse is $A = \pi ab = 21\pi$, and the force is $F = 84445 \text{ lb}$



Problem 10.109 In Problem 10.108, determine the magnitude of the net moment exerted on the window about the horizontal axis L by the pressure of the seawater ($\gamma = 64 \text{ lb/ft}^3$) and the atmospheric pressure of the air on the opposite side. (See Problem 10.89.)

Solution: The moment is

$$M = \int px \, dA = \gamma \int x^2 \, dA = \gamma I_y.$$

The area moment of inertia for an ellipse is

$$I_y = \frac{\pi ab^3}{4} = 202 \text{ ft}^4,$$

and the moment is $M = \gamma I_y = 12930.8 \text{ ft lb}$.

Problem 10.110* The gate has water of 2-m depth on one side. The width of the gate (the dimension into the paper) is 4 m, and its mass is 160 kg. The mass density of the water is $\rho = 1000 \text{ kg/m}^3$ and atmospheric pressure is $p_{\text{atm}} = 10^5 \text{ Pa}$. Determine the reactions on the gate at A and B. (The support B exerts only a horizontal reaction on the gate.)

Solution: Consider the free-body diagram shown. The weight of the gate is

$$W = (160)(9.81) = 1570 \text{ N},$$

$$\text{and } x_2 = R - \frac{2R}{\pi} = 0.727 \text{ m}.$$

The pressure force

$$\begin{aligned} F &= \frac{1}{2}(\gamma R)(R)(4) \\ &= \frac{1}{2}(1000)(9.81)(2)^2(4) \\ &= 78,480 \text{ N}, \end{aligned}$$

$$\text{and } y = \frac{1}{3}R = 0.667 \text{ m}.$$

The area below the gate is

$$A_b = \frac{1}{4}\pi R^2 = 3.142 \text{ m}^2,$$

and the centroid of A_b is at

$$x_b = R - \frac{4R}{3\pi} = 1.151 \text{ m}.$$

The area above the gate is

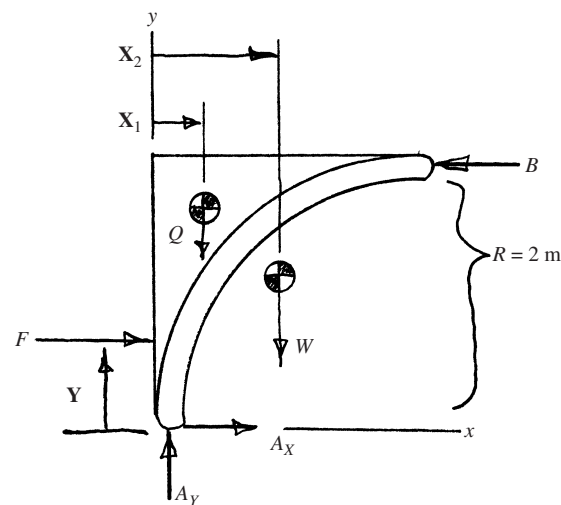
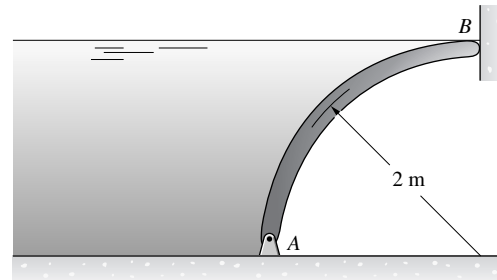
$$A_a = R^2 - A_b = 0.858 \text{ m}^2.$$

The centroid of $A_a \cup A_b$ is at $x = R/2$, so

$$\frac{R}{2} = \frac{x_1 A_a + x_b A_b}{A_a + A_b},$$

from which we obtain $x_1 = 0.447 \text{ m}$.

The weight of the water is $Q = \gamma A_a(4) = 33,700 \text{ N}$.



From the equilibrium equations

$$\sum F_x = F + A_x - B = 0,$$

$$\sum F_y = A_y - W - Q = 0,$$

$$\sum M_{(ptA)} = RB - yF - x_1 Q - x_2 W = 0,$$

we obtain

$$A_x = -44.2 \text{ kN},$$

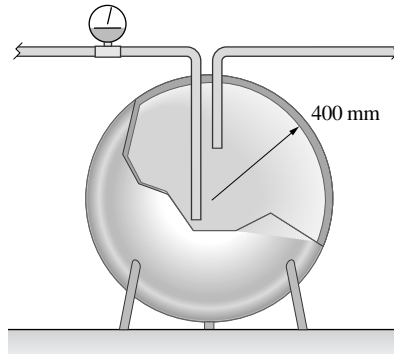
$$A_y = 35.3 \text{ kN},$$

$$B = 34.3 \text{ kN}.$$

Problem 10.111 A spherical tank of 400-mm inner radius is full of water ($\rho = 1000 \text{ kg/m}^3$). The pressure of the water at the top of the tank is $4 \times 10^5 \text{ Pa}$.

- What is the pressure of the water at the bottom of the tank?
- What is the total force exerted on the inner surface of the tank by the pressure of the water?

Strategy: For (b), draw a free-body diagram of the sphere of water in the tank.



Solution: The weight density is $\gamma = \rho g = 9810 \text{ N/m}^3$. ((a) The pressure distribution is $P(x) = p_0 + \gamma x$.) The pressure at the bottom of the tank is $P(0.8) = p_0 + \gamma (0.5) = 4 \times 10^5 + (9810)(0.8) = 4.0785 \times 10^5 \text{ Pa}$.

(b) From the free body diagram of the sphere of water, the unbalanced force is the weight of water, acting downward:

$$W = \left(\frac{4}{3}\right) \pi R^3 \gamma = 2629.9 \text{ N}$$